Dynamical behaviour of cyclic production lines under instationary conditions with a system dynamics approach

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Abstract

In this paper, we investigate the dynamical behaviour of cyclic production lines under the presence of stochasticity An approach is presented which allows the modeling of a large part of the production line as a single "module" which is characterized by a holistic description covering the effective dynamical behaviour by means of a system dynamics approach. To this aim, a module is viewed as a system which is described by two binary random variables describing the module's behaviour at its beginning and its end. These random variables are controlled by a state variable N(t) which denotes the mean number of items in the module.

The dynamics of the module is then given by a system dynamics approach where the change of stock equals the difference of incoming and outgoing flow. It is shown by a simulation study that this simple approximate model is able to reproduce correctly typical dynamical features of the filling-up process of a production line.

Keywords

Stochastic modeling, methodology, industrial production systems, automobile industry.

1 Introduction

In the age of globalization the demands on production lines with respect to productivity, cost efficiency, quality of output and flexibility are ever increasing. Many markets require competitors to offer a broad product range with shorter and shorter innovation cycles in order to secure an adequate market share and a profitable business strategy. Together with a more volatile customer demand this implies significant increases in the number of variants and in the model mix as well as markedly reduced product life cycles leading. Managing the complexity of highly dynamic production lines under these circumstances will be the key to sustainable profitability and success in the market place. Simulation has become more and more important in recent years as a tool to show the implications of decisions and the interdependency of different processes during both planning and operational phase of a production line. However, in order to be useful as a management decision support tool a simulation model needs to be simple and intuitive representing only the main interactions rather than the every detail of the actual system while still maintaining a high degree of accuracy.

Traditionally, manufacturing systems are modelled as a topologic structure of stations and buffers, handling items and being controlled by internal rules. However, often it is difficult to understand and interpret the system's dynamics due to the complex interactions between items, stations and buffers. The larger the system, the less this "atomic" bottom-up approach helps to understand the dynamical behaviour of the system. This paper presents a hierarchical modelling approach in which single stations or complete sections of production lines as well as the line itself can be described as modules that have a well determined dynamical behaviour.

Thus, the model is derived from a different perspective to manufacturing systems which is not based on an individual item view but rather involves some "macroscopic" or global concepts. In

the field of queueing networks, such new approaches have been developed based on a concept of fluid dynamics ([Chen and Yao 1992], [Chen and Mandelbaum,1994] , [Glynn 1990]). These concepts, too, are an attempt to replace the item focused description to more global quantities.

Often, a macroscopic description can be attained by stochastic models where stochastic elements are used for approximatively replacing complex processes and interactions by some sort of "noise". Thus, the model must be viewed as an approximation. Even though, however, such an approximation may be sufficient for solving an optimization task. In [Heitz 2003], a model for a cyclic production line has been described which is able to explain the stationary behaviour of an arbitrary large production line by means of a characteristic function which is a macroscopic description of the internal structure. It has been shown that, for realistic failure probabilities of the working stations, the results of the approximative model are very close to the ones for a full simulation model.

In the present contribution, the model is extended for the non-stationary case.

2 The model

We consider a part of a cyclic production line, consisting of a sequence of working stations and buffers which is called a *module*. A complete production line is assumed to consist of one or more coupled modules. The stations are modeled as Markovian elements: Each station may fail to perform its work on the item in a given production cycle with given failure probability. In this case, the item is processed in the next production cycle again. Thus, an item may stay at a work station for two or more subsequent periods.

In each production cycle, a module may receive an item from the upstream module, and may deliver an item to the downstream module. The dynamical behaviour of a module is assumed to be described by two binary stochastic variables X_{in} and X_{out}. X_{in} describes the ability of the module to accept an item, and X_{out} describes the ability of the module to deliver an item. For a sequence of production cycles t=0,1,2,…, a module is described by a sequence $X_{in}(t)$ and $X_{out}(t)$ which, as an approximation, is assumed to be an iid sequence [Heitz 2003]. In this framework, for each production cycle t, the module is described by the probability of being able to deliver (*delivery probability)* p(t) and the probability of being able to accept (*acceptance probability)* q(t) which are defined by

$$p(t) = P\left[X_{out}(t) = 1\right] \quad \text{and} \quad q(t) = P\left[X_{in}(t) = 1\right] \tag{1}$$

When coupling different modules together, the delivery and acceptance probabilities p_i and q_i, respectively, of a considered module i depend on the delivery probability p_{i-1} of the upstream module and the acceptance probability q_{i+1} of the downstream module. Thus, the probabilities of neighbouring modules are coupled, leading to characteristic functions $p_i(p_{i-1},q_{i+1})$ and $q_i(p_{i-1},q_{i+1})$, respectively. It has been shown in [Heitz 2003] that, for realistic failure probabilities of the working stations, this simple model can be used for calculating production rates with a high accuracy.

For extending the approach of [Heitz 2003] to instationary conditions, the time dependence of p_i and q_i has to be taken into account. For this, a generalized system dynamics approach is used. In system dynamics, a system is described by stocks and flows where the stocks represent the internal state of the system, and the flows are the generators which change this internal state [Sterman 2000]. Usually, a change of the stock is accomplished by the difference between input flow and output flow. The dynamics is determined by the laws which generate the input and output flows in dependence of the internal state of the system and the interactions to connecting systems.

For a production system, the stochasticity of the item flow has to be taken into account. Thus, the dynamical laws are described on a stochastic process level as follows: The input and the output

flows are described by the *expected* input and output item flows which are given by $p_{i-1} \cdot q_i$, and $p_i \cdot q_{i+1}$, respectively. Analogously, the internal state is characterized by the stock variable $N_i(t)$ which denotes the *expected* number of items in the system at cycle t. Note that this description of the internal state by means of a single number is a strong simplification which, for example, does not account for the distribution of the items within the module.

The dynamics of the system in this simplified framework is given by

$$dN_i(t) = p_{i-1}(t) \cdot q_i(t) - p_i(t) \cdot q_{i+1}(t) \tag{2}$$

where $dN_i(t)$ denotes the change of $N_i(t)$ between time step t and t+1:

$$dN_i(t) = N_i(t+1) - N_i(t) \cdot \tag{3}$$

The quantities $p_{i-1}(t)$ and $q_{i+1}(t)$ denote the external boundary conditions of the considered module *i*, while the quantities $q_i(t)$ and $p_i(t)$ describe the behaviour of the module itself.

We assume that $q_i(t)$ and $p_i(t)$ are functions of the boundary conditions and the state $N_i(t)$:

$$p_i(t) = p_i(p_{i-1}, q_{i+1}, N_i) \tag{4}$$

$$q_i(t) = q_i(p_{i-1}, q_{i+1}, N_i) \tag{5}$$

where the time index on the right-hand side is suppressed for the sake of simplicity.

The dynamics of the system is generated as follows: According to the current values of p_{i-1}, q_{i+1} and N_i, the acceptance and the delivery probabilities are set according to Eqs. (4) and (5). With these values, the change dN can be calculated according to Eq. (2), leading to an updated state parameter for time t+1.

There are two open questions:

1. Is this simple model able to reproduce the dynamical behaviour correctly?
2. What are the functional relationships Eqs. (4) and (5)?

For the first question, we focused on the case of filling up of an initially empty production line. To this aim, we compared the dynamical model as formulated above with a full simulation model.

The second question was solved partially by analyzing simulation results and interpolating them. In the present phase, no attempt was made to derive the functional relationship from the structural properties of the module.

3 Simulations

Numerous simulations of a production module consisting of three stations and three buffers were made [Engeler 2003]. The sequence of the elements is: buffer-station-buffer-station-buffer-station. The failure probability of the stations is set to a value of 0.1 or 0.2. All buffers have a size of 10. This leads to a maximum number N_{max}=33 items in the system (30 items in the buffers, and 3 items in the stations).

For the upstream boundary condition, the values p_{i-1}=1, 0.95, 0.9, 0.85 are used. The downstream boundary conditions are changed between q_{i+1}=0.1 and 0.8 in steps of 0.1. Each combination of p_{i-1} and q_{i+1} was simulated, leading to 4·8 =32 simulation experiments. For each simulation, the boundary conditions are chosen to be time-independent. Additionally, the structure module itself was the same for all simulation experiments.

Since the buffers are rather large, a good decoupling of the stations is achieved, leading to a maximum production rate of the module (under perfect boundary conditions p_{i-1}=1 and q_{i+1}=1) of nearly 0.8, or 0.9, respectively, parts per cycle. Thus, the largest value q_{i+1}=0.8 of the downstream module corresponds to a following production system with roughly the same

performance as the considered module. Lower values of q_{i+1} correspond to a following bottleneck, leading to a filling-up of the module to a value near N_{max}.

For each simulation, the initial condition is an empty module. 500 time steps were simulated which, in each case, was sufficient to reach the stationarity. For each simulation experiment, many independent simulation runs were performed. From these, the number $N_i(t)$ was estimated for each t=1,..,500 by averaging the observed number of items over all simulation runs. Analogously, the acceptance probability $q_i(t)$ and the delivery probability $p_i(t)$ are measured for given time t by averaging over all simulation runs.

As an example, in Fig. 1 (left) the estimated curve N(t) together with some typical trajectories of N(t). Note that although the number of items in the system is an integer number for a single trajectory, the expectation value N(t) is a real number. This leads to a smooth N(t)-curve.

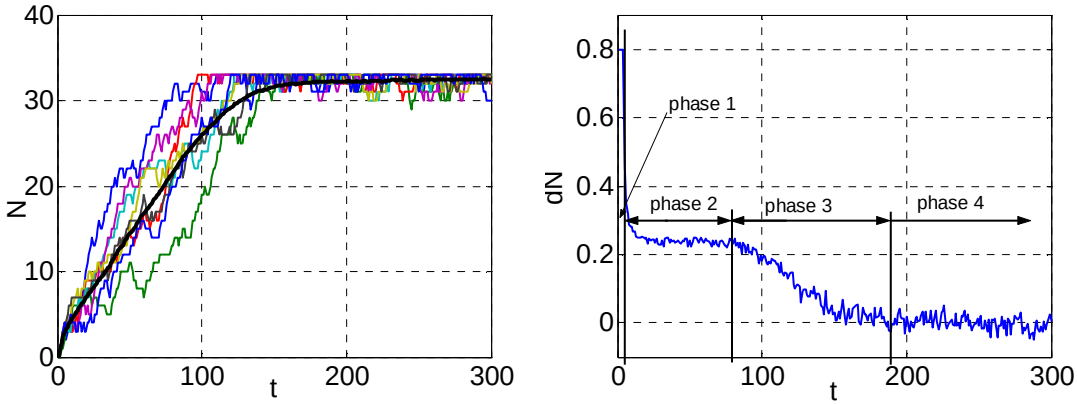

Figure 1: Left: Typical trajectories of the number of items for 7 independent simulation runs. Thick line: Average number of items N(t). Right: dN(t) for the same simulations. Simulation parameters: p_{i-1}=0.8, q_{i+1}=0.6, failure probability = 0.1.

The dynamics can be seen more clearly if the change of N(t) is regarded (see Fig.1 right).

In most simulations, four typical phases are observed: First, the mean number of items N(t) increases very rapidly (large dN) for few cycles. This corresponds to the initial transport of items: q_i is 1 because the first buffer is still empty, but p_i is zero because no items have reached the last station.

In the second phase, the number increases linearly (constant dN). This corresponds to an acceptance probability q_i of near 1 (first buffer not full in most simulation runs) and a constant delivery probability of the last station. Since the boundary conditions are constant, this leads to a constant dN and a filling up of the module.

In phase 3, the probability of the first buffer being full increases, leading to a reduced acceptance probability. This reduces the incoming flow dN until a value of dN=0 (phase 4), corresponding to the stationary condition: input flow equals output flow. Even if the theoretical dN must be exactly zero, the estimated dN from the simulation runs fluctuate around zero due to random errors.

4 Results for specified boundary conditions

In a first step, the approach was tested for fixed p_{i-1}=0.8, q_{i+1}=0.6, and failure probability of 0.1. In each time step, the quantities $N_i(t)$, $p_i(t)$ and $q_i(t)$ were estimated from the independent simulation runs.

Since, in Eqs. (4) and (5), all parameters of the functions $q_i(p_{i-1},q_{i+1},N_i)$ and $p_i(p_{i-1},q_{i+1},N_i)$ are constant except of $N_i(t)$, one may plot the values $q_i(t)$ and $p_i(t)$ against $N_i(t)$, leading to a single characteristic function $q_i(N)$ and $p_i(N)$, respectively, which describes the dynamical behaviour of

the module for all time steps. In Fig 2, each point corresponds to a pair ($N_i(t)$, $q_i(t)$) and ($N_i(t)$, $p_i(t)$), respectively. The time ordering of these points plays no role.

It can be seen that all points scatter around a line which can be identified with the desired functional relationships $p_i(t) = p_i(p_{i-1}, q_{i+1}, N_i)$ and $q_i(t) = q_i(p_{i-1}, q_{i+1}, N_i)$, respectively. The thick lines in Fig. 2 are polynomial interpolations of the simulation data. In the right picture, the used polynomials lead to oscillations which are fictitious and do not describe a real feature of the data.

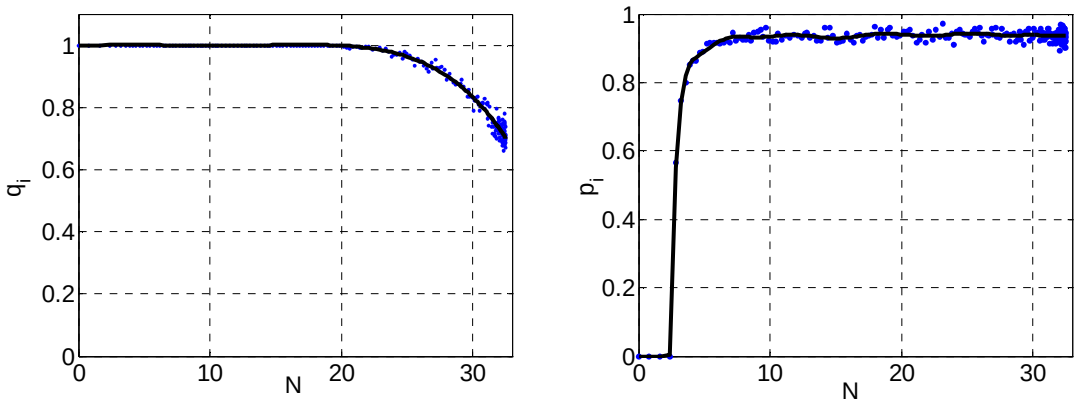

Figure 2: Characteristic function $p_i(N)$ and $q_i(N)$ of the module for fixed boundary conditions p_{i+1}=0.8 and q_{i+1}=0.6

These polynomial interpolations can be used for evaluating the dynamical equation (2) with the initial condition N(0)=0. This leads to a N(t)-behaviour as displayed in Fig. 3.

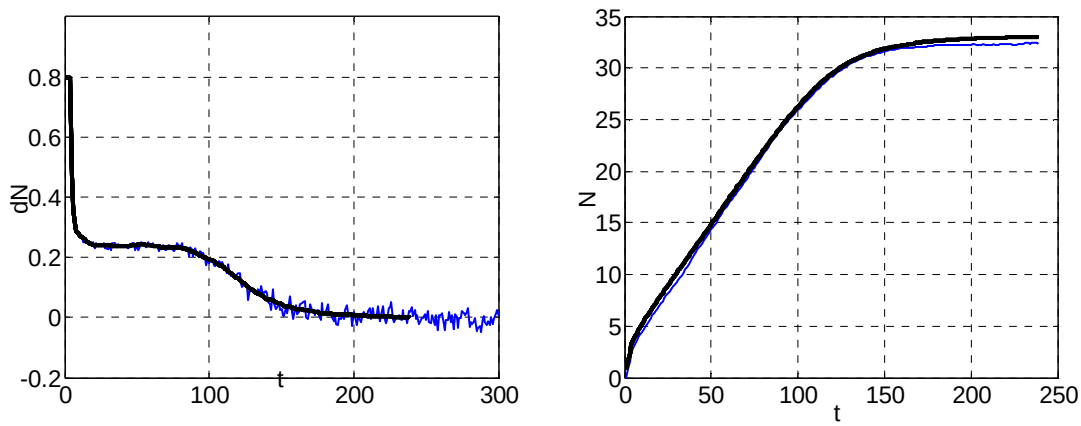

Figure 3: Reconstruction of dN(t) (left) and N(t) (right) from the basic dynamical equation (2) with interpolated characteristic function according to Fig. 3. Thick line: Calculation according Eq. (2). Thin line: Original data from simulation.

For this example, the system dynamics can be reconstructed correctly by means of the characteristic functions. This function is valid for all time points and for all four phases of the filling-up procedure and thus represents an integral description of the module.

Note, however, that the boundary condition was not changed. For each set of boundary conditions p_{i+1} and q_{i+1}, different functions $p_i(N)$ and $q_i(N)$ are obtained.

5 Results for arbitrary boundary conditions

In this section we investigate the approach for different values of boundary conditions as specified in Section 3. The aim is to find the characteristic functions which are valid for all

boundary conditions and all times. The failure probability of the stations was set to 0.2 for these simulations.

In Fig. 4, p_i and q_i are plotted against N for all parameter settings. The whole time range t=0,..,500 is used but not all data is displayed for the sake of readability. The different symbols indicate the downstream boundary: ·, x, o, + correspond to q_{i+1}=0.8/0.7, 0.6/0.5, 0.4/0.3, 0.2/0.1. Thus, each symbol denotes two different downstream conditions. The different upstream boundary conditions (p_{i-1}=1,0.95,0.9, and 0.85) are not distinguished in the plots and plotted with the same symbol.

The delivery probability p_i is nearly independent on N for N>5 but depends both on p_{i-1} and q_{i+1}. (the dependence on p_{i-1} cannot be seen in the figure). As an approximation, we set

$$p_i = \begin{cases} 0, & N < 5 \\ p_i(p_{i-1}, q_{i+1}), & N \geq 5 \end{cases} \tag{6}$$

ignoring the dependence on N for N≥5. The values of p_i for the different parameter settings (p_{i-1},q_{i+1}) are obtained from the stationary behaviour t→∞, similar as in [Heitz 2003].

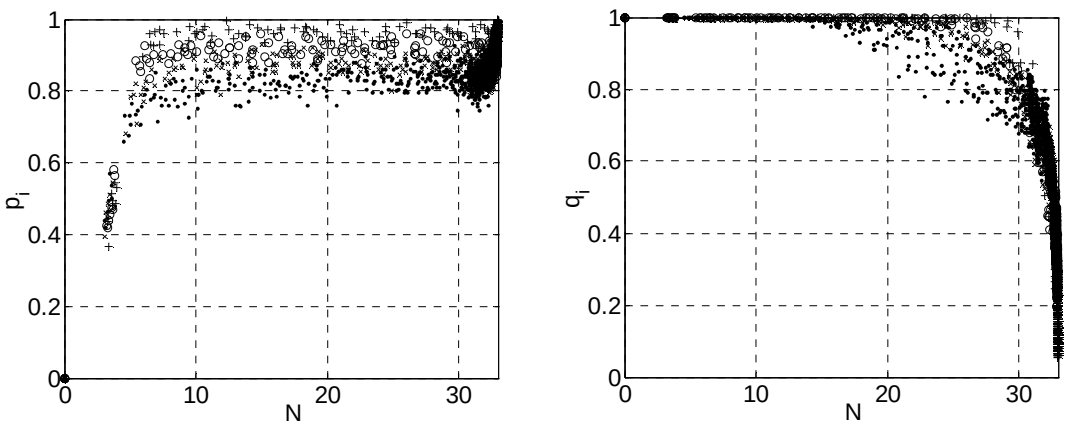

Figure 4: Dependence of p_i and q_i on N for all parameter settings. Left: p_i(N), Right: q_i(N). The symbols indicate the downstream boundary: ·, x, o, + correspond to q_{i+1}=0.8/0.7, 0.6/0.5, 0.4/0.3, 0.2/0.1. The different upstream boundaries cannot be distinguished.

In contrast to p_i, the acceptance probability q_i is mainly determined by N. It is independent on the boundary conditions for N<15 and N>30. The influence of the boundary conditions is visible in the range of N=15…30.

For N<15, we have q_i≈1. For determining the q_i-behaviour for N→N_{max}, in Fig. 5, q_i is plotted as a function of Δ=N_{max}-N. A logarithmic law can be verified for small Δ. The thick line is a linear regression to the semilogarithmic data where the data of all simulations have been used. This leads to the regression formula

$$q_i = 0.1266 \cdot \log(\mathrm{N}_{\max} - \mathrm{N}) + 0.6376 \tag{7}$$

Since q_i cannot be larger than 1, we set q_i=1 it the above formula gives a value larger than 1.

In Fig. 6 and Fig. 7, calculations of N(t) and dN(t), respectively, for two different parameter settings are shown. It can be seen that the dynamical behaviour of the module during the filling process is reproduced quite well. The four different dynamical phases are reproduced quantitatively.

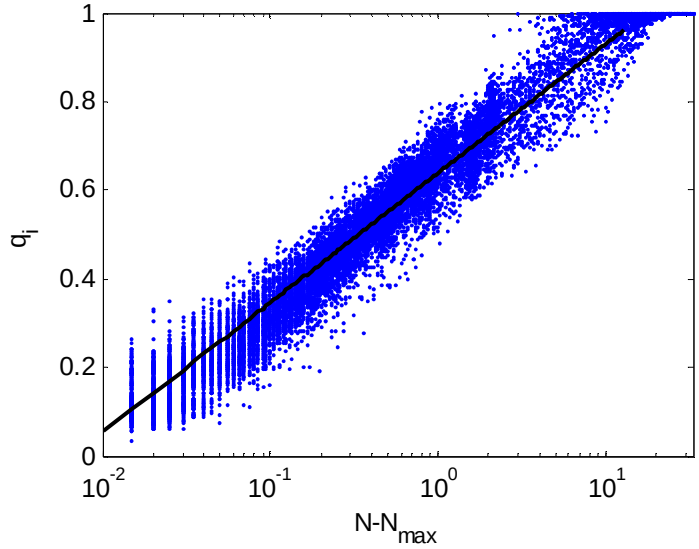

Figure 5: Plot of q_i against $\Delta=N_{max}-N$. All simulated data is plotted. The thick line is a linear interpolation of the semilogarithmic data according to Eq. (7).

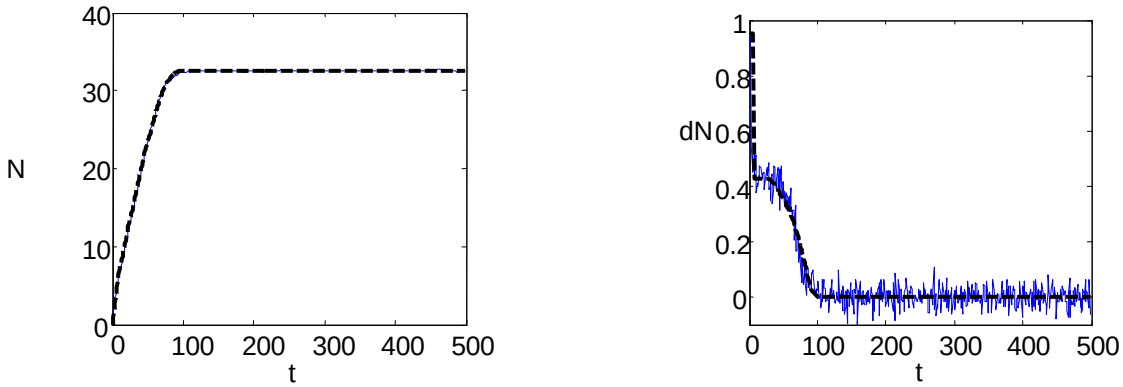

Figure 6: Reconstruction of N(t) (left) and dN(t) (right) for p_{i-1}=0.95, q_{i+1}=0.6. Thin line: Estimation from simulation. Thick line: Integration of dynamical model according Eqs. (2) to (5).

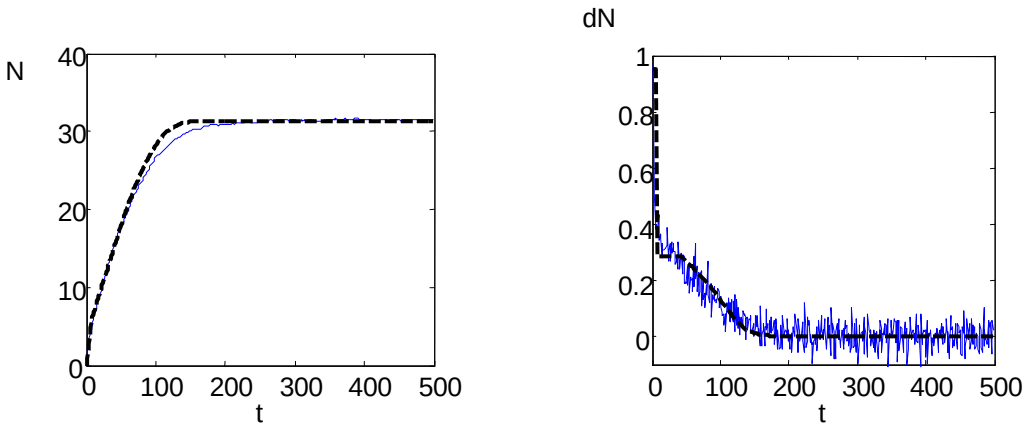

Figure 7: Reconstruction of N(t) (left) and dN(t) (right) for p_{i-1}=0.95, q_{i+1}=0.8. Thin line: Estimation from simulation. Thick dashed line: Integration of dynamical model according Eqs. (2) to (5).

6 Conclusion

We have investigated a simple stochastic model for cyclic production lines for the description of the dynamical behaviour of a module consisting of several working stations and buffers which is coupled to an upstream module with delivery probability p_{i-1} and an downstream module with acceptance probability q_{i+1}. The model is based on a generalized system dynamics approach with a stock variable N(t) denoting the expected number of items in the module at time t. The temporal change of N(t) is generated by the difference between expected incoming and outgoing flow which results from the boundary conditions and the probabilities p_i and q_i of the considered module.

For a given module with three stations and three buffers, it could be shown for a wide range of boundary conditions that the main dynamical features of the filling process of the module can be reproduced by characteristic functions $p_i(p_{i-1},q_{i+1},N)$ and $q_i(p_{i-1},q_{i+1},N)$. The different phases of the filling process are well reproduced by the integration of the basic dynamical equation. It must be noted, however, that the results still are preliminary. The approach has to be tested in a larger range for external boundary conditions as well as for instationary boundary conditions. Furthermore, the functional dependence of the characteristic functions on the parameters p_{i-1}, q_{i+1} and N is not yet clarified.

However, it seems that the above developed framework for describing the dynamical behaviour of production lines by means of characteristic functions is able to capture the main dynamical features, thus giving a possibility for interpreting and understanding the dynamical behaviour in a macroscopic view.

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