Parameter tables for PID controllers for time delayed systems optimized with a learning method

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Abstract. This publication provides useful parameter sets in tabular form for PID controllers for various rise and dead times of step responses of asymptotically stable control systems, which minimize the common quality criteria in the time domain, integral of absolute error IAE, integral of time-multiplied absolute value of error ITAE and integral of sqared error ISE. Since the determination of the parameter sets is very computationally intensive, an approach from the field of artificial intelligence was chosen.

The application of the parameter sets found is verified with examples. The parameter sets also take into account the controller output limitations that are relevant in practice and can basically be used for all PID controllers of controlled systems with a time delay.

1 Introduction and related work

PID controllers are still by far the most frequently used controller structures for single-in, single-out (SISO) systems. The control of controlled systems with dead time is challenging. The parameters found with heuristic methods lead to aysmptotically stable systems. The most famous of them are those from Ziegler Nichols, Latzel, or Chien Hrones and Reswick. However, all parameters found with these methods still have to be readjusted in the practical system so that sensible transient behavior results.

The time-delayed systems are very common in practice. They require special demands, because their control is challenging. In practice, however, they are very common, especially in process engineering or in thermal systems, since the sensor often can not be placed directly next to the actuator.

There are different approaches known for finding PID controller parameters from step responses of time-delayed systems. All of them result in stable control systems. Especially, as the dead time increases, it becomes difficult to find suitable PID parameter sets. There are some heuristic methods for this, which can be used in the time and frequency domain. However, these parameters must be further optimized afterwards. The first approach was the parameter set from Ziegler Nichols [1]. There are also several others existing, for example Chien, Hrones and Reswick [2]. For further optimization, there are several methods used, also some from the field of artificial intelligence, for example particle swarm optimization, PSO [3].

In this paper, the method hill climbing [4] is used. It is another stochastic method for optimizing controllers, but it is related to PSO. For optimizing controller parameters, there also other approaches known [5]–[9], [13]–[15].

In order to be able to handle time-delayed systems in terms of simulation at all, the turning point tangent method is often used. A PTn system is identified with n PT1 (1st order) elements connected in series, which have identical time constants. They are dealt with in the literature [10], [11], [12]. The PID parameter tables, which are described and used in the next chapters, however, refer to these PTn systems with identical time constants. Such systems are very common and can be found in all engineering disciplines. The series connection of such PTn systems according to formula (1) leads to step responses which are delayed. In particular, the dead times can be approximated with linear models in this way. Here, Ks is the static gain, n is the system order and T1 is the time constant of the n identical PT1 elements.

$$\frac{K_s}{(s \cdot T_1 + 1)^n} \tag{1}$$

2 Identification of PTn

The turning point tangent should be used here as a reference for identification. In many cases, one can simply measure the delay time Tu and the rise time Tg according to figure 1 by placing a tangent at the point of inflection. From this one can identify the number n of PT1 elements connected in series and their identical time constants T1.

The measurement of the step response of a controlled system can then be dealt with using table 1, which is well known from literature [10].

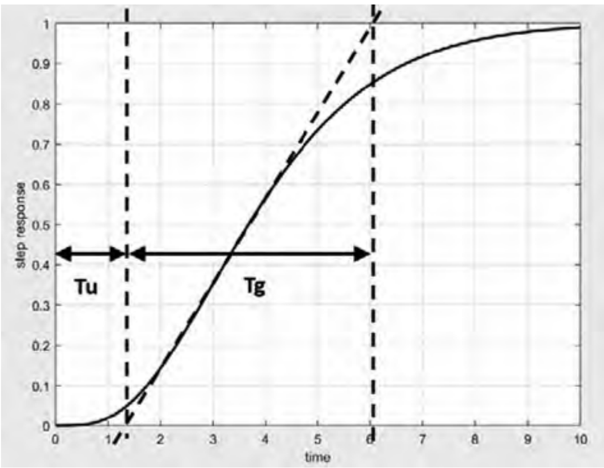

Figure 1: Step response of a PTn- Systems, turning point tangent and subdivision into Tu and Tg.

n, PTn	Tg/T1	Tu/T1	Tg/Tu
2, PT2	2.72	0.28	9.65
3, PT3	3.69	0.81	4.59
4, PT4	4.46	1.43	3.13
5, PT5	5.12	2.10	2.44
6, PT6	5.70	2.81	2.03

Table 1: Calculation of Tg, Tu, T1 and PTn

The parameters in table 1 can be calculated using formulas 2 to 4, for different system order n.

$$\frac{T_g}{T_1} = \frac{(n-1)!}{(n-1)^{n-1}} \cdot e^{n-1} \quad (2)$$

$$\frac{T_u}{T_1} = n - 1 - \frac{(n-1)!}{(n-1)^{n-1}} \cdot [e^{n-1} - \sum_{m=0}^{n-1} \frac{(n-1)^m}{m!}] \quad (3)$$

$$\frac{T_g}{T_u} = \frac{\frac{(n-1)!}{(n-1)^{n-1}} \cdot e^{n-1}}{n - 1 - \frac{(n-1)!}{(n-1)^{n-1}} \cdot [e^{n-1} - \sum_{m=0}^{n-1} \frac{(n-1)^m}{m!}]} \quad (4)$$

3 ITAE, IAE and ISE criterions

The block diagram of the controlled system is shown in figure 2. The parameters found for the PTn system are Ks, T1 and n.

Among others, the criteria IAE, ITAE and ISE are used for optimizing, which describe the error area of a step response of the controlled system. These error areas are shown in figure 3.

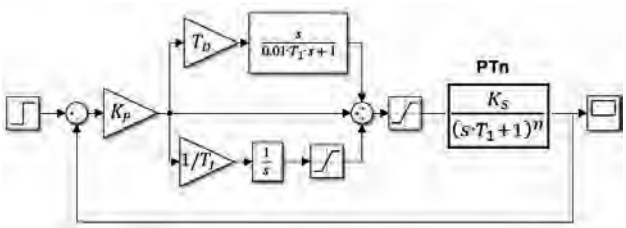

Figure 2: Block diagram of a PTn- systems, which is controlled by a PID controller.

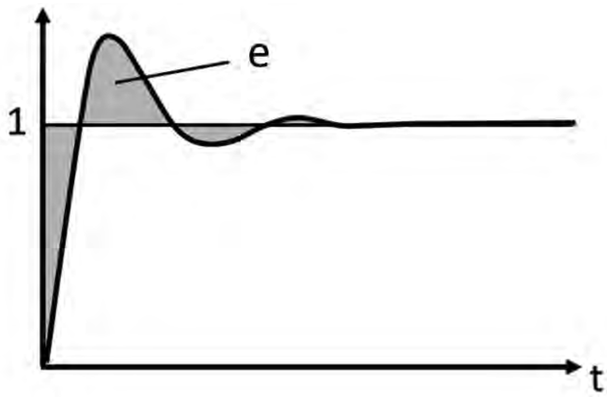

Figure 3: Error area in the transient response of the closed-loop system according to figure 2, for calculating the IAE, ITAE and ISE criterions.

IAE means integral of absolute error, ITAE means integral of time-multiplied absolute value of error and ISE means integral of sqared error. It can be seen from this that the IAE criterion calculates the amount of the error area. The ITAE criterion is an extension of the IAE criterion and also takes time into account. Thus, the error area is weighted more heavily as time progresses. The two criteria IAE and ITAE are also called the L1 criterion. The ISE criterion does not calculate the error area, but its square. This means that there it is no need to calculate the absolute values of the error area, since negative signs cancel each other out when squared. The ISE criterion is also called the L2 criterion.

$$IAE\colon \int_0^\infty |e(t) - e(\infty)|\, dt \tag{5}$$

$$IITAE\colon \int_0^\infty |e(t) - e(\infty)| \cdot t \cdot dt \tag{6}$$

$$ISE\colon \int_0^\infty [e(t) - e(\infty)]^{\,2}\, dt \tag{7}$$

4 Calculation of PID parameters with the hill climbing learning method

Now, since in chapter 1 the time-delayed systems were approximated with the turning point tangent method as PTn elements, the quality criteria for step responses calculated for all orders n and also all PID controller parameters Kp, Ti and Td. The parameters which correspond to the minimum of the criteria can then be displayed as table values. The problem is that this has to be done for a multidimensional space (order n, Kp, Ti, Td, quality criteria). So it would take far too long with the computing power available today. Therefore, ‘hill climbing’, a method from the field of artificial intelligence was chosen [4]. With this method, heuristic functions are added to some of the parameters, in this case the parameters of the PID controller, and then it is calculated whether the quality criteria IAE, ITAE and ISE have become smaller. If there is, the new parameters will be used as a reference. If not, the old ones stay. In this way and after many iterations, the final values of the parameters remain at local minima of the quality criteria. The method requires much less computing time than a complete calculation in multi-dimensional space, for example with nested loops of all parameters. With the 5 parameters, order n, Kp, Ti, Td, quality criteria, this would have the time complexity f (n) = O (x5).

However, since it only finds local minima, several different random tuples of starting values for the control parameters are used. Since many of the results of the converged parameters for the minimal quality criteria then agree with one another, it can be assumed with reasonably good certainty that the parameters found are actually PID parameters Kp, Ti and Td, which either correspond to the absolute minima of the criterions, or which come very close to these at least.

The search for optimal parameters in multi-dimensional space, as with this specific problem in control engineering, is also one of the good arguments for using an artificial intelligence method here as well. Often, complete calculations cannot be carried out in the entire parameter space due to the computing power available. Since only part of the parameter space is calculated with such methods, the computing time is significantly reduced and the results are parameter sets for excellent transient behavior.

5 PID controller parameters after the minimization of the quality criterions IAE, ITAE and ISE

This chapter is the essence of the publication. The table below describes the PID controller parameters calculated with Matlab / Simulink and the ‘hill climbing’ method according to the minimized quality criteria IAE, ITAE and ISE. The block diagram in figure 2 serves as the basis for this. It is particularly noteworthy that the static gain Ks and in particular the time constant T1 of the n identical PT1 elements are included in the table. This makes them usable and scalable for all PTn systems. The values up to n = 6 are shown here. The controller output limitation is implemented on the one hand after the controller and on the other hand also after the integrator (anti windup) and is assumed to be +/- 2, +/- 3, +/- 5, +/- 10. In the calculations the anti windup is never active, but it is inserted anyway, because in practice it can happen for various reasons that the controlled variable does not reach the desired variable in the static end value.

The controller output limitation is calculated as (maximum possible controller output - controller output before the step) divided by (controller output for the stationary end value - controller output before the step). In many cases, the controller output before the step ist equal to 0, thus the controller output limitation is calculated as maximum possible controller output divided by controller output for the stationary end value. In the table, the maximum parameter value is limited to 10.

PT1	**+/- 2**	**+/- 3**	**+/- 5**	**+/- 10**
IAE	Kp·Ks = 10 Ti = 3.1·T1 Td = 0 (PI)	Kp·Ks = 10 Ti = 2·T1 Td = 0 (PI)	Kp·Ks = 10 Ti = 1.3·T1 Td = 0 (PI)	Kp·Ks = 10 Ti = 1·T1 Td = 0 (PI)
ITAE	Kp·Ks = 9.3 Ti = 2.9·T1 Td = 0 (PI)	Kp·Ks = 9.5 Ti = 1.9·T1 Td = 0 (PI)	Kp·Ks = 9.1 Ti = 1.2·T1 Td = 0 (PI)	Kp·Ks = 10 Ti = 1·T1 Td = 0 (PI)
ISE	Kp·Ks = 10 Ti = 2.7·T1 Td = 0 (PI)	Kp·Ks = 10 Ti = 1.6·T1 Td = 0 (PI)	Kp·Ks = 9.8 Ti = 1.5·T1 Td = 0 (PI)	Kp·Ks = 10 Ti = 0.2·T1 Td = 0 (PI)

PT2	**+/- 2**	**+/- 3**	**+/- 5**	**+/- 10**
Tg/Tu: 9.65				
IAE	Kp·Ks = 10 Ti = 9.6·T1 Td = 0.3·T1	Kp·Ks = 10 Ti = 7.3·T1 Td = 0.3·T1	Kp·Ks = 10 Ti = 5.6·T1 Td = 0.3·T1	Kp·Ks = 10 Ti = 3.7·T1 Td = 0.2·T1
ITAE	Kp·Ks = 10 Ti = 9.6·T1 Td = 0.3·T1	Kp·Ks = 10 Ti = 7.3·T1 Td = 0.3·T1	Kp·Ks = 9.6 Ti = 5.4·T1 Td = 0.3·T1	Kp·Ks = 9.8 Ti = 4.7·T1 Td = 0.3·T1
ISE	Kp·Ks = 10 Ti = 9.7·T1 Td = 0.2·T1	Kp·Ks = 10 Ti = 7.3·T1 Td = 0.2·T1	Kp·Ks = 10 Ti = 5.1·T1 Td = 0.2·T1	Kp·Ks = 10 Ti = 4.6·T1 Td = 0.1·T1

PT3	**+/- 2**	**+/- 3**	**+/- 5**	**+/- 10**
Tg/Tu: 4.59				
IAE	Kp·Ks = 5.4 Ti = 9.4·T1 Td = 0.7·T1	Kp·Ks = 7 Ti = 10·T1 Td = 0.7·T1	Kp·Ks = 8.4 Ti = 9.8·T1 Td = 0.7·T1	Kp·Ks = 10 Ti = 9.7·T1 Td = 0.7·T1
ITAE	Kp·Ks = 5.4 Ti = 9.4·T1 Td = 0.7·T1	Kp·Ks = 7 Ti = 10·T1 Td = 0.7·T1	Kp·Ks = 8.2 Ti = 9.6·T1 Td = 0.7·T1	Kp·Ks = 10 Ti = 9.7·T1 Td = 0.7·T1
ISE	Kp·Ks = 6.1 Ti = 10·T1 Td = 0.6·T1	Kp·Ks = 8.1 Ti = 9.8·T1 Td = 0.6·T1	Kp·Ks = 10 Ti = 10·T1 Td = 0.6·T1	Kp·Ks = 10 Ti = 7.8·T1 Td = 0.6·T1

PT4	**+/- 2**	**+/- 3**	**+/- 5**	**+/- 10**
Tg/Tu: 3.13				
IAE	Kp·Ks = 2 Ti = 5.2·T1 Td = 1.1·T1	Kp·Ks = 2.9 Ti = 6.5·T1 Td = 1.2·T1	Kp·Ks = 3.3 Ti = 7.1·T1 Td = 1.3·T1	Kp·Ks = 3.3 Ti = 6.9·T1 Td = 1.3·T1
ITAE	Kp·Ks = 1.9 Ti = 5·T1 Td = 1.1·T1	Kp·Ks = 2.4 Ti = 5.9·T1 Td = 1.2·T1	Kp·Ks = 2.3 Ti = 5.7·T1 Td = 1.2·T1	Kp·Ks = 2.1 Ti = 5·T1 Td = 1.1·T1
ISE	Kp·Ks = 2.8 Ti = 6.6·T1 Td = 1.2·T1	Kp·Ks = 3.6 Ti = 7·T1 Td = 1.2·T1	Kp·Ks = 4.9 Ti = 7.1·T1 Td = 1.4·T1	Kp·Ks = 5.2 Ti = 7·T1 Td = 1.4·T1

PT5	**+/- 2**	**+/- 3**	**+/- 5**	**+/- 10**
Tg/Tu: 2.44				
IAE	Kp·Ks = 1.7 Ti = 5.8·T1 Td = 1.6·T1	Kp·Ks = 1.8 Ti = 5.9·T1 Td = 1.6·T1	Kp·Ks = 1.8 Ti = 5.8·T1 Td = 1.6·T1	Kp·Ks = 1.7 Ti = 5.5·T1 Td = 1.6·T1
ITAE	Kp·Ks = 1.4 Ti = 5.3·T1 Td = 1.4·T1	Kp·Ks = 1.4 Ti = 5.2·T1 Td = 1.4·T1	Kp·Ks = 1.4 Ti = 5.2·T1 Td = 1.4·T1	Kp·Ks = 1.4 Ti = 5.0·T1 Td = 1.4·T1
ISE	Kp·Ks = 1.9 Ti = 5.9·T1 Td = 1.7·T1	Kp·Ks = 2.6 Ti = 6.5·T1 Td = 1.8·T1	Kp·Ks = 2.5 Ti = 6.3·T1 Td = 1.8·T1	Kp·Ks = 2.5 Ti = 6.1·T1 Td = 1.8·T1

PT6	**+/- 2**	**+/- 3**	**+/- 5**	**+/- 10**
Tg/Tu: 2.03				
IAE	Kp·Ks = 1.3 Ti = 5.9·T1 Td = 1.9·T1	Kp·Ks = 1.3 Ti = 5.8·T1 Td = 1.9·T1	Kp·Ks = 1.3 Ti = 5.8·T1 Td = 1.9·T1	Kp·Ks = 1.3 Ti = 5.6·T1 Td = 1.9·T1
ITAE	Kp·Ks = 1.1 Ti = 5.5·T1 Td =1.7·T1	Kp·Ks = 1.1 Ti = 5.5·T1 Td = 1.7·T1	Kp·Ks = 1.1 Ti = 5.4·T1 Td = 1.7·T1	Kp·Ks = 1.1 Ti = 5.3·T1 Td = 1.7·T1
ISE	Kp·Ks = 1.8 Ti = 6.8·T1 Td = 2.1·T1	Kp·Ks = 1.8 Ti = 6.5·T1 Td = 2.1·T1	Kp·Ks = 1.8 Ti = 6.5·T1 Td = 2.1·T1	Kp·Ks = 1.8 Ti = 6.3·T1 Td = 2.1·T1

Table 2: Table values of the PID parameters for the minimum IAE, ITAE and ISE criterions of controlled PTn or time delayed systems.

It is noteworthy that the table scales with T1 and Ks. The results are therefore very widely applicable.

6 Applications of table values

6.1 Control of a PT3 system

In the first application example, a didactic example is used to show the general usability of the parameter table. The response of a time-delayed system to a unit jump shows a static end value of 1, a delay time Tu of 0.81 seconds and a rise time Tg of 3.69 seconds. This results in Tg / Tu = 4.59 and this results in a PT3 behavior with Ks = 1 and T1 = 1 second.

For the ITAE criterion, the table values of the PID parameters for the PT3 system are read off. Since Ks = 1 and T1 = 1s, the table values are multiplied by 1 and therefore correspond to those for the controller parameters Kp, Ti and Td. The simulation of the step responses of the closed loop system according to figure 2 is shown in figure 4. It shows a very nice transient response. The different dynamics or rise times can be explained with the different controller output limitations. This also shows very well that these must be included into the controller design.

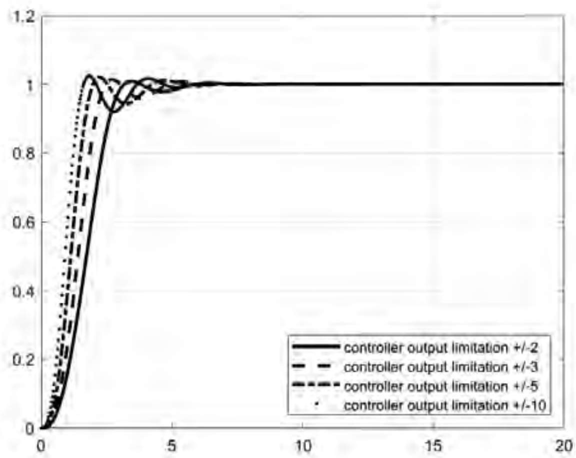

Figure 4: Step response of the closed loop system according to figure 2 for a PT3 system with the PID table values for the ITAE criterion.

6.2 Control of a PT2 system, comparison with Ziegler Nichols and Chien, Hrones and Reswick

In the following, the controller parameters found are compared with those of Ziegler-Nichols and Chien, Hrones and Reswick, using an example of second order. The used system has an order n = 2 and a time constant T1 of 8s. As a comparison to practical systems this could be a thermal system, where the heating coil and the temperature sensor are not exactly located at the same place.

$$\frac{K_s}{(s \cdot T_1 + 1)^n} = \frac{1}{(s \cdot 8 + 1)^2} \tag{8}$$

The controller output signal for the stationary end value of the controlled system is 1 and the controller output signal is limited to +/- 2, which results in a controller output limitation factor +/- 2. Using the table 1, it results for PT2 a Tg = 21.76 seconds and Tu = 2.24 seconds

According to the Ziegler-Nichols step response method, controlled systems with dead time and a PT1 are treated. In this case, Tu is assumed to be the dead time and Tg as the time constant. This results in the controller parameters:

$$Kp = \frac{1.2 \cdot Tg}{Ks \cdot Tu} = 11.65, \quad Ti = 2 \cdot Tu = 4.48s$$

$$Td = 0.5 \cdot Tu = 1.12s$$

According to Chien, Hrones and Reswick with the parameters for 'aperiodic', the result is:

$$Kp = \frac{0.6 \cdot Tg}{Ks \cdot Tu} = 5.83, \qquad Ti = 1.0 \cdot Tg = 21.76s$$

$$Td = 0.5 \cdot Tu = 1.12s$$

The method calculated above with the parameters according to the minimal ITAE criterion provides a T1 of 8s and n = 2 according to Table 1, i.e. a PT2 behavior. This results in the following parameters from table 2:

$$Kp = \frac{10}{Ks} = 10, \quad Ti = 9.6 \cdot T1 = 76.8s,$$

$$Td = 0.3 \cdot T1 = 2.4s$$

The simulation according to the block diagram according to figure 1 (PT2 with PID) shows the results according to figure 5 for the three parameter sets.

The rise time is similar for all three parameter sets, because all systems run into the controller output limitation in this phase. It shows very nicely that the calculated values with the minimum ITAE criterion according to Table 2 show an excellent transient behavior.

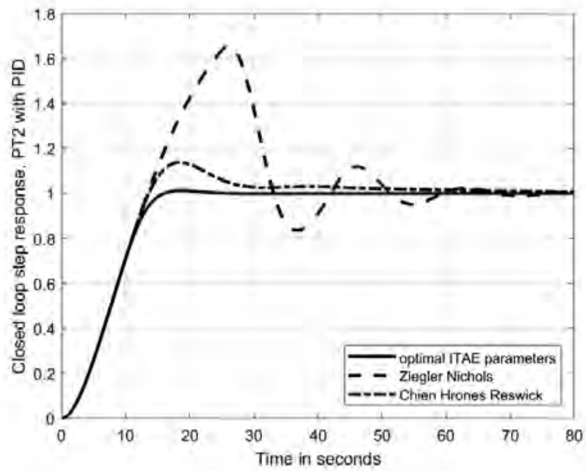

Figure 5: Comparison of the step responses with the control parameters for a PT2 plant, with a control output limitation factor +/- 2.

6.3 Control of a general PT4 system, comparison with Ks ≠ 1 and T1 ≠ 1

In the next example, a general problem is dealt with in order to also show the scalability of the presented para- -meter table. There was measured the response to a unit step and it is shown in figure 6.
With the application of the turning point tangent method according to figure 1 and table 1, the step response leads to the following transfer function, with Ks = 0.4 and T1 = 0.5s:

$$G = \frac{0.4}{(s \cdot 0.5 + 1)^4} \quad (9)$$

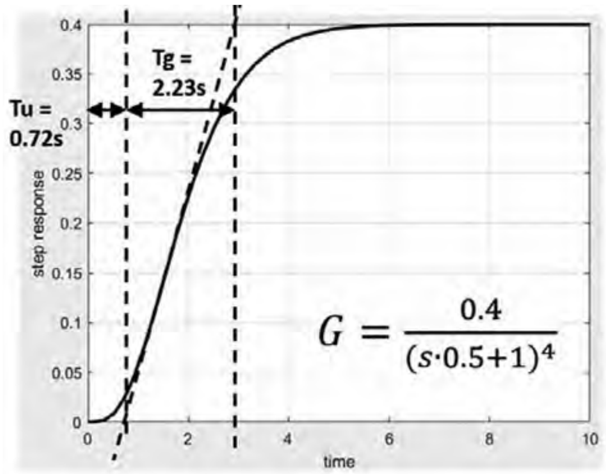

Figure 6: Step response of a PT4 system.

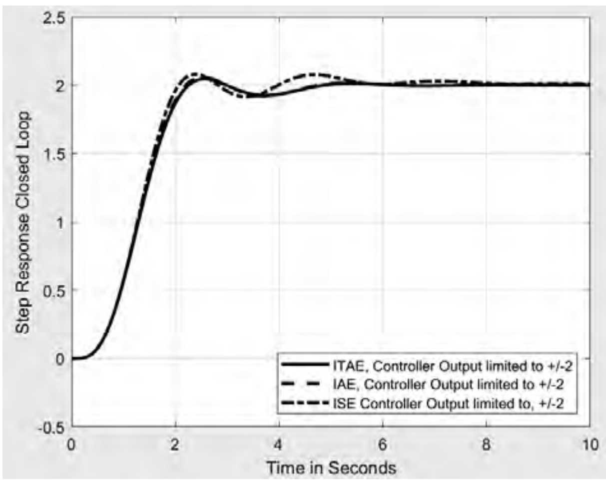

Figure 7: Response of a closed loop, PID with PT4 for a step from 0 to 2

One would like to design the system with a PID controller according to figure 2 and execute a setpoint jump from 0 to 2. Since the static gain Ks = 0.4, the controller output for the stationary end value is then 2 / Ks, or 2 / 0.4 = 5. Assuming that the controller output is limited to +/- 10, the result is a controller output limitation factor +/- 10/5 = +/- 2.
The controller parameters are to be calculated for the IAE criterion as an example.

$$Kp = \frac{2}{Ks} = 5 \quad Ti = 5.2 \cdot T1 = 2.6s$$

$$Td = 1.1 \cdot T1 = 0.55s$$

For minimizing the criteria IAE, ITAE and ISE, this results in the closed loop behavior according to figure 7 for a setpoint jump from 0 to 2.

7 Discussion and outlook

Good transient behavior can be seen for all parameter sets in the table. Compared with heuristic methods, these parameters are hard-calculated values that minimize the quality criteria. It is also up to the discussion what would happen if one would performe different jumps and therefore had to choose the parameters according to different factors of the controller output limitation. The parameters are very similar, however, and values for jumps should be selected which are most likely to occur in the specific system. Even for the general setpoint jump by any value, the parameters still give very good transient behavior.
An exciting finding emerges from the discussion of the question of how the parameters Kp, Ti and Td develop for changing ratios Tg / Tu (i.e. rise time in relation to the delay time).
The smaller the ratio, the greater the delay time in relation to the rise time, the smaller Kp on the one hand and the greater Td on the other hand.
The effect of a small Kp means that the system can only be regulated slowly. In the literature [10] this is also described in such a way that the controllability for systems with longer dead times is reduced. If one were to also plot the manipulated variable, one would see that this is also only relatively small. Therefore it is of no use in these systems if an additional regulator reserve is made available through amplifiers, because this cannot be used at all due to the time delay of the system.
If you follow the development of the value of Kp in the

tables, then with smaller ratios Tg / Tu (or larger orders n of the PTn systems and thus larger dead times) and smaller Kp, greater system dynamics are no longer achieved. On the one hand, the controlled variable shows a nice transient response according to the minimized quality criteria and, in particular, also reaches the setpoint in the stationary end value, which is often sufficient in practice.

When looking at the differential component of the controller Td, it becomes apparent that a differential component for optimizing the quality criteria is missing when regulating frequently occurring PT1 elements, i.e. a pure PI controller is already optimal. With an increasing system order, i.e. a decreasing ratio Tg / Tu or a larger delay, the required D component (Td) becomes larger and larger.

It turns out that the PTn systems that occur very frequently in practice can be regulated very well with the table values available according to the minimized IAE, ITAE and ISE criteria. In practice, you can often do without a simulation and only measure the step response of the system. Then the order n and the associated parameters for the PID controller can be read from the table, also using Ks and T1 and implement the controller directly on the system.

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