A Systematic Solution to the Bridge and Torch Riddle

Combinatorics can be used to derive the solution to the bridge and torch riddle. The approach is more precise than simple guessing and more intuitive than purely formal solutions (e.g. an algebraic solution).

The classic bridge riddle involves four people who need to cross a bridge as quickly as possible under certain specific conditions: 1) The bridge can carry a maximum of only 2 persons at any given time. 2) Given that it is night, the bridge can only be crossed with a torch. There is only one torch available, meaning that it will need to be carried back and forth. 3) Whenever 2 people attempt to cross at the same time, they can only go as fast as the slower of the two. 4) The four people take the following amounts of time [in minutes] to cross the bridge: 1, 2, 5, and 10.

The shortest amount of time (X minutes) in which all four people can cross the bridge under the given conditions is usually specified. The task is to determine: "In what order must the people cross, so as to keep within time X?"

Generalisation of the task

A partial generalisation of the task requires one to ascertain the shortest possible amount of time it would take to cross the bridge. The task can be solved using combinatorics. While the steps required increase disproportionately as the number of people involved increases, they remain manageable in our example with only four people. Furthermore, one could also avail oneself of general algorithms for any number of parameters such as number of people, times, etc. (cf. References).

Step 1: Parameters and process

In order to succeed, the following parameters must be minimised:

- Number of crossings (minimum no. = 5);
- "Lost"time on the way across or back.

The crossings can be represented as a process: three ways across and to ways back (minimum).

Step 2: Pairs and their times

The term combinatorics in the present context refers to the creation of (complete) variant trees and the ruling out of variants on the basis of logical deductions or relative comparisons (prioritisation). It could also be described as systematic trial and error.

The people are assigned their crossing time as an identification number. There are 6 possible combinations ranging from A to F, with 1+2 and 5+10 exhibiting the best ratio of lost time to definitive time.

Step 3: Combinations for the way out

Theoretically, there are 41 possible combinations. Only five of these are really possible, however, because all of the others would involve either a missing person (~~XYZ~~) or a slow person going across twice (~~XYZ~~). At 14 minutes, AAF is the best option.

Step 4: Combining ways out and ways back

Of the remaining three variants AAF, AFA and FAA, only AFA is possible, although the two ways back could be reversed. The time required is 17 minutes. ABC, etc. are rendered superfluous.

1) Process [number of people]

Crossing	Start	Way	Goal
No. 0	4		0
No. 1	2	2 >	2
No. 2	3	< 1	1
No. 3	1	2 >	3
No. 4	2	< 1	2
No. 5	0	2 >	4

2) Pairs and times [minutes]

Comb.		Def.	Lost	Ratio
A	**1 + 2**	2	-1	**- 0.5**
B	1 + 5	5	-4	- 0.8
C	1 +10	10	-9	- 0.9
D	2 + 5	5	-3	- 0.6
E	2 +10	10	-8	- 0.8
F	**5 +10**	10	-5	**- 0.5**

3) Variant tree for way across

~~AAA to FFF~~

~~AAB to AAE~~

AAF 14 Min.

~~BBC to BBF~~ ~~CCD to CCF~~

~~DDE and DDF~~ ~~EEF~~

4) Variant tree for ways out and back

Start	Way	Goal	Time
1 2 5 10	1 2 >	1 2	2
	< 1		1
1 5 10	5 10 >	2 5 10	10
	< 2		2
1 2	1 2 >	1 2 5 10	2

ABC 17 Min. ~~ABD~~

ABE 17 Min. ~~ABF~~

ACD 17 Min. ~~ACE~~ ~~ACF~~

ADE 17 Min. ~~ADF~~ ~~AEF~~

~~BCD~~ ~~BCE~~ ~~BCF~~ ~~BDE~~ ~~BDF~~ ~~BEF~~

~~CDE~~ ~~CDF~~ ~~CEF~~ ~~DEF~~

References

Rote, Günter (2002): Crossing the Bridge at Night, Freie Universität Berlin, Institute of Computer Science. shows the algebraic solution, reviews earlier works.

Backhouse, Roland (2008): Capacity C Torch Problem, University of Nottingham. reduces the task to a calculation of the shortest way.

variant tree slow person going across twice

way

out >	A 1+2: 3*3*2=18 possibilities																	
< back	1									2								
out >	B 1+5			C 1+10			F 5+10			D 2+5			E 2+10			F 5+10		
< back	1	2	5	1	2	10	2	5	10	1	2	5	1	2	10	1	5	10
out >	C 1+10	E 2+10		B 1+5	D 2+5		A 1+2			C 1+10	E 2+10		B 1+5	D 2+5		F 1+2		
time	**19 Min.**	**20 Min.**		**19 Min.**	**20 Min.**		**17 Min.**			**20 Min.**	**21 Min.**		**20 Min.**	**21 Min.**		**17 Min.**		
comb.	**ABC**	**ABE**		**ABC**	**ACD**		**AFA**			**ACD**	**ADE**		**ABE**	**ADE**		**AFA**		

out >	B 1+5: 3*3*2=18 possibilities																	
< back	1									5								
out >	A 1+2			C 1+10			E 2+10											
< back	1	2	5	1	5	10	2	5	10									
out >	C 1+10	E 2+10		A 1+2			A 1+2											
time	**19 Min.**	**20 Min.**		**19 Min.**			**20 Min.**											
comb.	**ABC**	**ABE**		**ABC**			**ABE**											

out >	C 1+10: 3*3*2=18 possibilities																	
< back	1									10								
out >	A 1+2			B 1+5			D 2+5											
< back	1	2	10	1	5	10	2	5	10									
out >	B 1+5	D 2+5		A 1+2			A 1+2											
time	**19 Min.**	**20 Min.**		**19 Min.**			**20 Min.**											
comb.	**ABC**	**ACD**		**ABC**			**ACD**											

out >	D 2+5: 3*3*2=18 possibilities																	
< back	2									5								
out >	A 1+2			C 1+10			E 2+10											
< back	1	2	5	1	5	10	2	5	10									
out >	C 1+10	E 2+10		A 1+2			A 1+2											
time	**20 Min.**	**21 Min.**		**20 Min.**			**21 Min.**											
comb.	**ACD**	**ADE**		**ACD**			**ADE**											

out >	E 2+10: 3*3*2=18 possibilities																	
< back	2									10								
out >	A 1+2			B 1+5			D 2+5											
< back	1	2	10	1	5	10	2	5	10									
out >	B 1+5	D 2+5		A 1+2			A 1+2											
time	**20 Min.**	**21 Min.**		**20 Min.**			**21 Min.**											
comb.	**ABE**	**ADE**		**ABE**			**ADE**											

F 5+10: 3*3*2=18 possibilities																	
5									10								

Total 108 possibilities   26 of which get all persons across   resulting in 5 types of combinations: AAF, ABC, ABE, ACD, ADE