

## Fuel Economy as Function of Weight and Distance

Fuel burn is crucial to the efficiency of aircraft operation. Empirical analysis shows, that it takes on average ~ 0.2 kg fuel to transport 1 kg of weight over a distance of 1'000 km. It also takes an additional ~ 0.02 to 0.03 kg fuel per 1'000 km for every kg of weight added. This is known as the marginal fuel burn rate (MFB). The opposite is also true: a reduction in weight by one kg saves ~ 0.02 to 0.03 kg of fuel per 1'000 km or ~ 2 to 3 €cent respectively.

To reduce the cost per available seat kilometre CASK of ~ 6 €cent by 1 ‰, the weight needs to be reduced by ~ 300 to 1'000 kg (structural or operational), depending on the aircraft type.

The optimum range is between ~ 2'000 km and 5'000 km for A322, 7'000 km for A333 and 12'000 km for A388. For shorter or longer distances, operation is less efficient because of fuel burnt for climb and the trade-off between fuel, payload and distance due to limitations of aircraft structure.

### 1. Subject of discussion and methodology

#### Empirical analysis and theoretical background

This article deals with marginal fuel burn as a function of weight or distance as well as the effects on aspects of fuel economy for commercial aircraft, such as specific fuel burn per pax\*distance [kg fuel/pax\*100 km] and cost per available seat kilometer CASK [€cent/av. seat \* km].

Empirical analyses of fuel burn are based on operational flight plans, calculated using simBrief [1] (referred to as OFP<sub>sim</sub>). Given the following basic assumptions, predictable results in the right scale are provided as a basis for understanding typical relations and functions.

Figures and functions are calculated using ~ 10 to 15 data points for approximation. Simplifications are justified by known theoretical concepts.

#### Parameters – ceteris paribus

Variation includes aircraft type according to its range and load capacity\*, trip distance (500 to 15'000 km) and payload (zero to maximum). All other parameters are defined within a typical range and no variation, following the concept of ceteris paribus. Operating speed is either held constant or optimised for long-range cruise LRC.

Assumptions include given cruising altitude (36'000 feet ASL), no winds aloft and no extra fuel for taxi, alternate, contingency or similar. Assumed average weight of passengers is 105 kg/pax, including 25 kg for luggage.

* air-craft	(referred to as)	range [km]	operat. speed [M]	accuracy of database
A320-200	(A322)	6'850	0.78	high
A330-300	(A333)	11'750	0.82	high
A380-800	(A388)	15'200	0.85	average

fig. 1 Aircraft types and accuracy of database [1].

Note: in physical terms “mass” is constant, while “weight force” as function of gravitation is not.

## 2. Fuel economy

### Performance ability of the airline industry

Services of the airline industry in 2012:

- volume of system [\$] \* 1'000'000'000'000
- volume of sales [\$] [3] 550'000'000'000
- available seat km ASK [2] 7'000'000'000'000
- passengers pax [2] 3'100'000'000
- flights [2] 32'000'000
- served airports [2] 2'500

\* estimated, see paragraph below.

Average numbers: seats available: ~ 125; pax per flight: ~ 100 and occupancy rate: ~ 80 %.

### Cost structure: an airline/system perspective

Cost per available seat kilometer CASK varies from 4 to 10 €cent [4]. Lufthansa LH (e.g.) states 8.8 €cent CASK for the year 2014 (6.7 €cent excluding fuel) [5]. 106 million pax travelled an average distance of 2'000 km per flight. Low CASK may result from optimised network and aircraft rotation, higher occupancy rate and lower customer service level amongst other.

CASK is from an airline perspective only. To understand the costs from a systems perspective, costs of ground infrastructures have to be taken into account: airport, (rail-)roads to the airports i.a.

share of costs [%]	CASK	LH	tCASK
operation	33	25	20
labour and depreciation	33	25	20
fuel	33	25	20
fees, taxes and provisions		25	20
additional infrastructure* & subsidies			20
<b>Total [€cent]</b>	<b>6</b>	<b>8</b>	<b>10</b>

fig. 2 Share of costs for CASK and tCASK (rounded numbers for network airlines).

10 €cent for tCASK seems to be a fair guess as a basis for showing the effects of weight reduction on the costs of the whole system, while the share of fuel is estimated at 2 €cent with a price of 1 € per 1 kg of fuel.

### Marginal fuel burn and costs

In business economics the cost of production of  $x$  units is divided into fixed (step-)costs (per production line or similar) and variable costs per piece, hereinafter presumed as constant.

$$C = \text{step fixed cost} + \text{variable costs per unit} \cdot x$$

The specific costs SC per piece decrease with the number of produced pieces until the capacity of the production line is reached and step-costs for a second production line have to be taken into account.

$$SC = \frac{\text{step fixed cost}}{x} + \text{variable costs per unit}$$

The marginal costs MC per piece are equal to the variable costs per piece.

$$MC = \text{variable costs per unit}$$

The function of marginal costs is the gradient (hence the derivation) of the costs as a function of produced units. Hence, for constant marginal costs the cost function is linear.

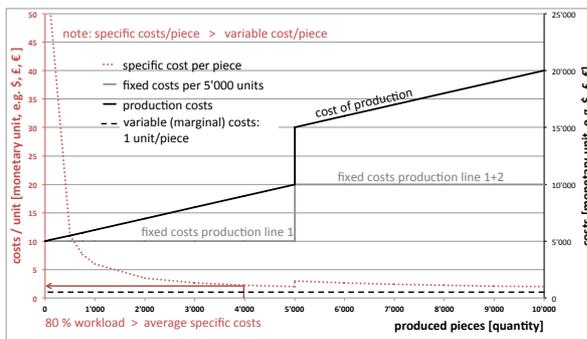


fig. 3 Cost of production and specific cost for variable (marginal) costs = constant (example).

To understand costs of production in fuel economy, they can be divided into

- fuel burn due to an aircraft with zero payload flying a given distance (= fixed step-costs),
- fuel burn per “produced piece” (= var. costs).

“Produced pieces” can be understood as transported payload [kg] (for a given distance) or travelled distance [km] (for a given weight).

### 3. Impact of weight - fuel burn as $f$ (payload)

#### Calculation of lift

$$\text{Lifted Weight } W = \frac{C_L \cdot \rho \cdot A \cdot v^2}{2 \cdot g} \cdot C \quad [\text{kg}]$$

Lift Coefficient  $C_L$  Area  $A$  [m<sup>2</sup>] true speed  $v$  [m/s]  $g = 9.8 \text{ m/s}^2$

$\rho$ : density as function of altitude and depth [kg/m<sup>3</sup>]

$C$ : correction factor for ground / submerge effect [6].

#### Angle of attack a.o.a.

All flying craft use lift produced by a.o.a. for most or all of the time that they are airborne. Additional lift is produced by the cambered shape of airfoils. A.o.a. of the fuselage floor is approximately zero for modern aircraft in cruise condition.

Lifted weight is proportional to a.o.a.:

$$a.o.a. = \frac{C_L}{0.11 \cdot e} = \frac{W \cdot 2 \cdot g}{\rho \cdot A \cdot v^2 \cdot 0.11 \cdot e}$$

$e$ : span efficiency factor =  $f$  (3D-Design and aspect ratio)

A.o.a. varies from zero to max. payload by approximately one degree for modern commercial aircraft.

air-craft	payload max. [to]	wing area [m <sup>2</sup> ]	delta a.o.a. [°]	vertical offset [m]	vertical offset* [h.h.]°
A322	17.2	123	<b>1.30</b>	0.85	1.0
A333	47.7	362	<b>1.10</b>	1.23	0.5
A388	93.9	846	<b>1.06</b>	0.72	0.25

\* vertical offset from nose to tail of the aircraft, for max. payload and delta a.o.a. respectively in [m] and for 1 kg payload in [h.h.]°

° [h.h.] = diameter of a human hair (50  $\mu\text{m}$ )

fig. 4 Variation of a.o.a. for zero to max. payload at cruising speed and altitude (without effect of additional fuel needed).

The increase of a.o.a. due to max. payload - at cruise speed and altitude - can be illustrated by the vertical offset from nose to tail (fig. 4). Additional rise of the nose is necessary to compensate for the weight of the additional fuel needed.

#### Lift and drag coefficients as function of a.o.a.

At low angles of attack the drag coefficient is low and small changes in a.o.a. create only slight changes in drag coefficient. At higher a.o.a. the drag coefficient (would be) much greater and small changes in angle of attack (would) cause significant changes in drag. [7]

For lift-drag ratio  $L/D$ , the maximum occurs at one specific a.o.a. and  $C_L$ . Modern aircraft are designed to fly in cruise condition at relatively high  $C_L$ .

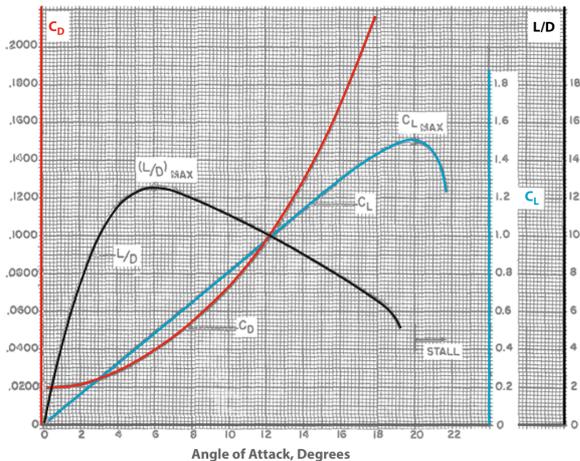


Figure 1.13. Drag Characteristics (sheet 2 of 2)

fig. 5 Drag characteristics, typical values (from [7], visually enhanced for better legibility).

**Power required as function of weight**

The primary effect of a weight change is a change in the induced drag and induced power required at any given speed. [7]

The curves of thrust required and power required provide the basis for comprehensive analysis of all the major items of airplane performance. The changes in the drag and power curves with variations of airplane gross weight, configuration and altitude provide insight [in terms of] the variation of range, endurance, climb performance etc., with these same [parameters]. (see fig. 6)

In an inverse view, change of weight (would for example) require the airplane to operate at different airspeeds to maintain conditions of a specific lift coefficient and a.o.a.

A.o.a. also defines the vertical and horizontal components of thrust, but changes are very small: ~ 1 % vertically (giving the aircraft an extra lift) and less than - 1 ‰ horizontally (decrease of thrust) for an increase in a.o.a. by 1° deg.

**Effect of weight on thrust and power required**

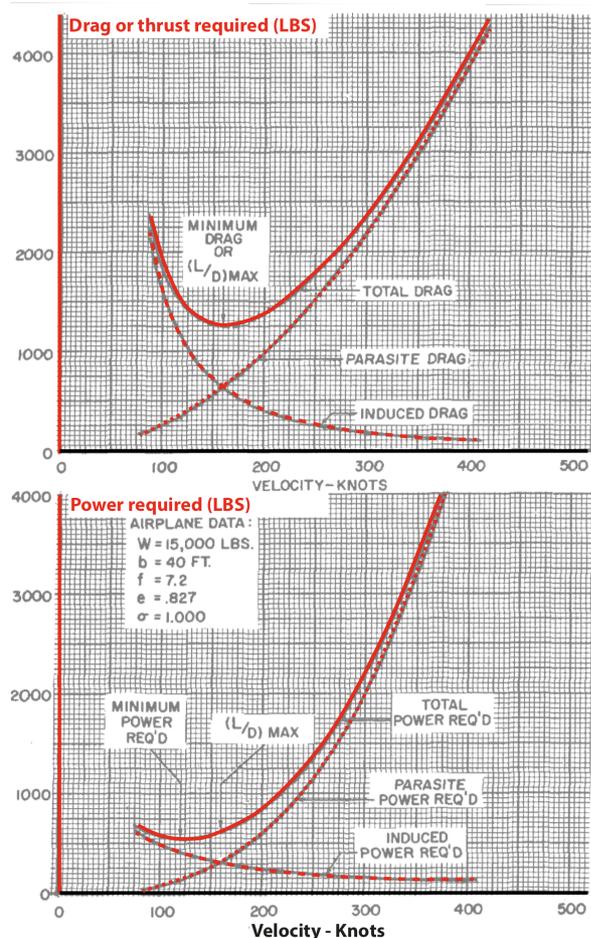


Figure 2.1. Airplane Thrust and Power Required

fig. 6ab Velocity vs. drag or thrust/power required/ (from [7], visually enhanced for legibility)

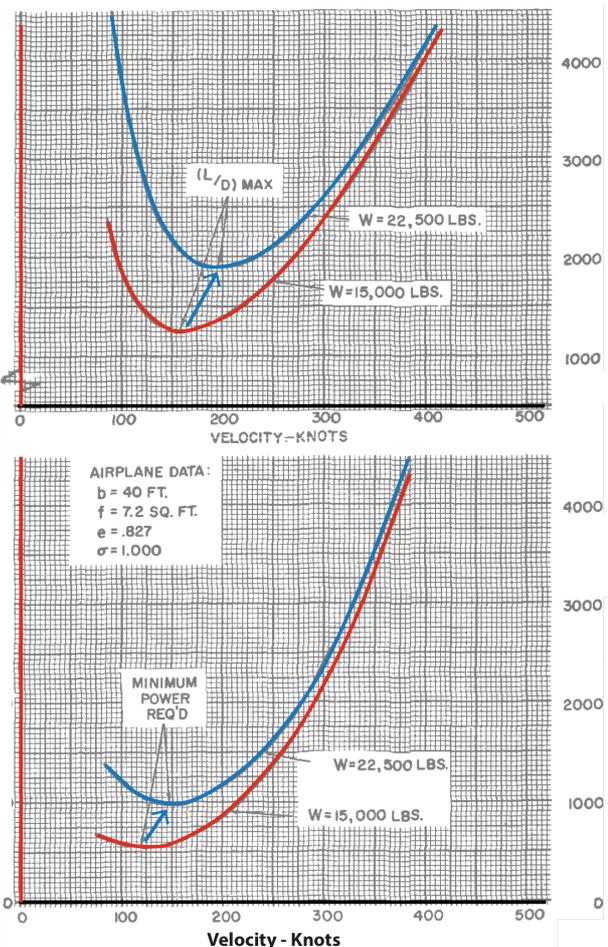


Figure 2.2. Effect of Weight on Thrust and Power Required

Effect of weight on thrust and power required (increase of weight to 150 %, ceteris paribus).

### Fuel burn as function of payload

The results of  $OFF_{sim}$  (see paragraph 1) for a given air distance (~ 4'200 km) show the basic shapes of curves. This distance lies within the optimum range for all 3 aircraft types (see paragraph 4, fig. 11).

Fuel consumption increases (approximately) linearly with aircraft weight, because weight corresponds to required thrust and thrust corresponds to fuel consumption. Fig. 5 (paragraph 3): shows the almost linear increases of  $C_D$  as a function of a.o.a. (as a function of payload).

In other words, idealization leads to constant marginal fuel burn and therefore fuel trip as a linear function of payload:

$$fuel\ trip\ [kg] = ft_{p,0} + MFB_{payload} \cdot payload\ [kg]$$

Maximum ratio of fuel trip due to payload is

$$\frac{ft_{p,max}}{ft_{p,0}} \sim 1.17\ to\ 1.37\ for\ air\ distance\ of\ 4'200\ km$$

Thus ~ 20 % of fuel burn is due to payload, while the rest is due to minimal take of weight  $TOW_{min}$ .

Marginal fuel burn due to payload is on average:

$$MFB_{payload} = \frac{\Delta fuel\ trip\ [kg]}{\Delta payload\ [kg]} \sim 8\ to\ 17\ %$$

A reduction of weight (structural or operating) by 100 kg results in a fuel saving of ~ 8 to 17 kg.

Fig. 7b shows only slight differences for the gradient of fuel trip as a function of payload. E.g. one fully loaded A388 burns the same amount of fuel as two A333 for the same trip (with 2 times 50 % of the payload of an A388).

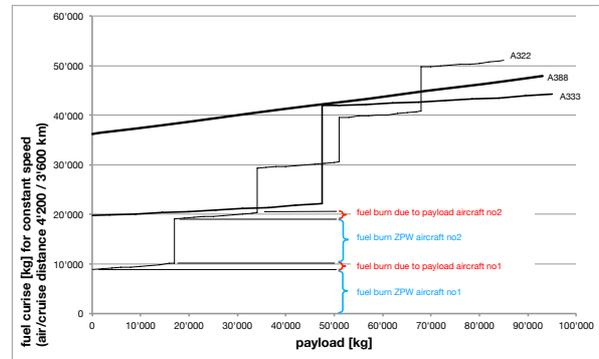


fig. 7 b) fuel trip for several aircraft of the same type as function of payload.

Specific Fuel Burn SFB as function of payload is similar for all three aircraft studied: 2.1 to 2.6 kg fuel / pax · 100 km. SFB is often referred to as "fuel efficiency", while Marginal Fuel Burn MFB as a function of payload differs by the factor 2 (see fig. 7a) and 7c): 2.0 to 4.1 kg fuel / 100 kg · 1'000 km.

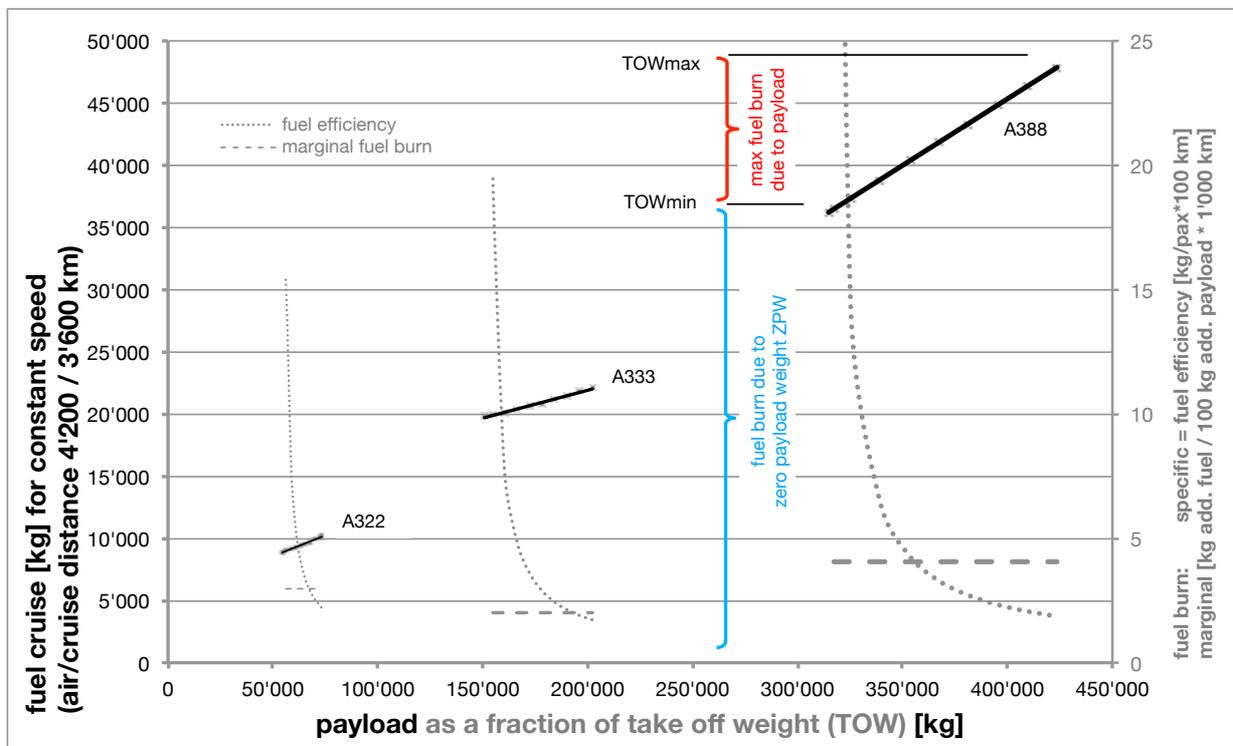


fig. 7 Impact of payload on fuel burn for a given air distance of ~ 4'200 km:

- a) payload as a fraction of takeoff weight, thus fuel trip for zero to maximum payload (left scale); Specific Fuel Burn SFB [kg fuel/pax · 100 km] (right scale)
- Marginal Fuel Burn MFB [kg add. fuel / 100 kg add. payload · 1'000 km] (right scale)

air-craft	payload max. [to]	occupied seats [number]*	add. FB [to]	$SFB_p$ $\left[ \frac{kg \text{ fuel}}{pax \cdot 100 \text{ km}} \right]$
A322	17.2	130	2.1	2.6
A333	47.7	364	4.0	2.1
A388	93.9	713	15.8	2.2

\* avg. occupancy rate ~ 80 % of max. seating as function of max. payload.

air-craft	$ft_{p,0}$ [to]	$MFB_{p, 4'200}$ $\left[ \frac{kg \text{ fuel}}{kg \text{ payload}} \right]$	$MFB_p$ $\left[ \frac{kg \text{ fuel}}{100 \text{ kg} \cdot 1'000 \text{ km}} \right]$	$ft_{p,max}$ [to]	$\frac{ft_{p,max}}{ft_{p,0}}$ [%]
A322	10.5	<b>0.12</b>	2.8	12.6	120
A333	23.4	<b>0.08</b>	2.0	27.5	117
A388	43.3	<b>0.17</b>	4.1	59.1	137

\* specific fuel burn for avg. occupancy rate (~ 80 %).

fig. 7 c) specific fuel burn ( $SFB_p = \text{Fuel Efficiency}$ ).

d) zero payload, average marginal fuel burn ( $MFB_p$ ) and max. payload.

CASK as a function of payload is obviously also a function of the number of available seats. Additional or saved expenditures are approximately

2.0 to 4.1 €cent per 1'000 kg payload \* km  
or per kg payload \* 1'000 km

$$\Delta CASK = \frac{\Delta \text{cost of fuel} \left[ \frac{\text{€cent}}{kg \text{ payload} \cdot km} \right]}{\text{available seats [number]} \cdot CASK \left[ \frac{\text{€cent}}{\text{seats} \cdot km} \right]}$$

$$\Delta \text{cost of fuel} = \frac{MFB \left[ \frac{kg \text{ fuel}}{kg \text{ payload}} \right] \cdot \text{price} \left[ \frac{\text{€cent}}{kg \text{ fuel}} \right]}{\text{trip distance } d \text{ [km]}}$$

air-craft	available seats [av. number]	$\Delta \text{cost of fuel} \left[ \frac{\text{€cent}}{kg \text{ payload} \cdot km} \right]$	$\Delta CASK_p \left[ \frac{\%}{kg p.} \right]$
A322	150	$2.8 \cdot 10^{-3}$	$3.2 \cdot 10^{-3}$
A333	277	$2.0 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$
A388	544	$4.1 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$

fig. 8 Impact of weight on cost of fuel and on  $CASK_p$  for a price of 1€/kg fuel.

This corresponds to 1.2 to 3.2 ‰ per 1000 kg. Thus, to reduce CASK by 1 ‰ for a flight of a given distance of 4'200 km, the weight of the aircraft (structural or operating) must be reduced by ~ 300 to 800 kg. This example may illustrate the scale of weight reduction needed to achieve significant economic savings.

## 4. Impact of distance

### Division of distance

Air distance can be divided into

- climb ~ 130 to 250 km
- cruise max. 5'000 / 11'000 / 15'000 km
- descent ~ 140 to 240 km

Max. cruise distance is given for A322/A333/A388.

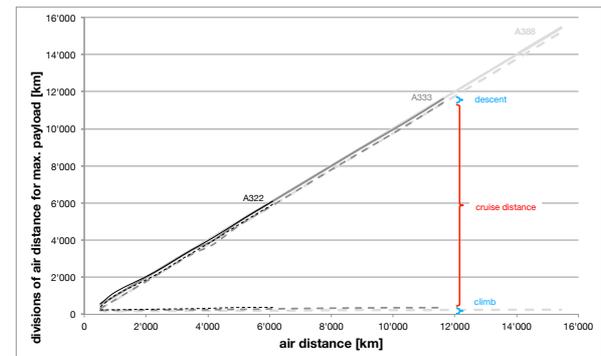


fig. 9 Division of air distance for max. payload.

The heavier the aircraft, the longer the distances for climb, while descent is usually performed at a fairly standard rate.

### Division of fuel burn

air-craft	climb [to]	cruise [to]	descent [to]	climb + descent [to]	trip [to]
A322	1 - 2	up to 16	0.2	1 - 2	2 - 19
A333	3 - 6	up to 71	0.5	3 - 6	4 - 78
A388	3 - 16	up to 223	2 - 2.5	5 - 18	7 - 240

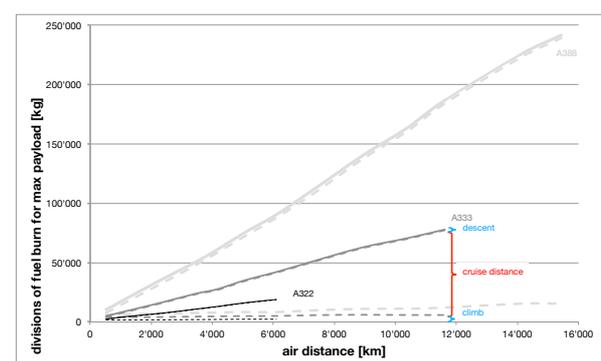


fig. 10 Division of fuel burn for max. payload.

The heavier the aircraft, the more fuel is burnt for climb, while the impact of weight on fuel burn for descent is negligible. For any trip distance greater than about 1/3 of the range of any aircraft analysed, more than 90 % of fuel is burnt during cruise.

### Fuel burn as a function of distance

The results of  $OFF_{sim}$  for zero to max. payload allow the following interpretations (see fig. 11). Fuel efficiency differs for short, optimum and long-range flight:

- for short distance flights, a significant amount of fuel is burnt for climb;
- for flights within an optimum range of distance, fuel efficiency is almost independent of trip distance: 2.1 to 2.6 kg fuel/pax\*100 km (see paragraph 3, fig. 7c); optimum range is from ~ 2'000 to 5'000 / 7'000 / 12'000 km;
- for long distance flights, the trade-off between fuel, payload and range leads to higher fuel burn / payload as well, thus fuel efficiency is poorer. This effect can also be seen on the curves for fuel trip as a function of air distance: A388, zero payload / max. payload (fig. 11).

The effect of the trade-off between range and payload is obviously dominant over the "fuel for fuel" effect (for long distance flight).

### Limitations for long distance flight

For long distances, structural limits are the reason for trade-offs between fuel, payload and range. Therefore, the curves for max. payload in fig 11 bend downwards in the long-distance range, resembling the shape of the curve in fig. 12.

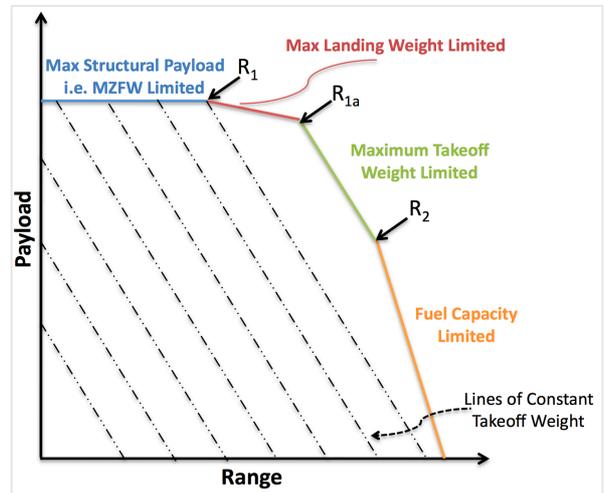


fig. 12 Notional payload-range diagram with trade-offs for Range > R<sub>1</sub> (from [8]).

R<sub>1</sub> - R<sub>2</sub>: trade-off between fuel and payload;  
R<sub>2</sub> - max.: trade-off between payload and range.

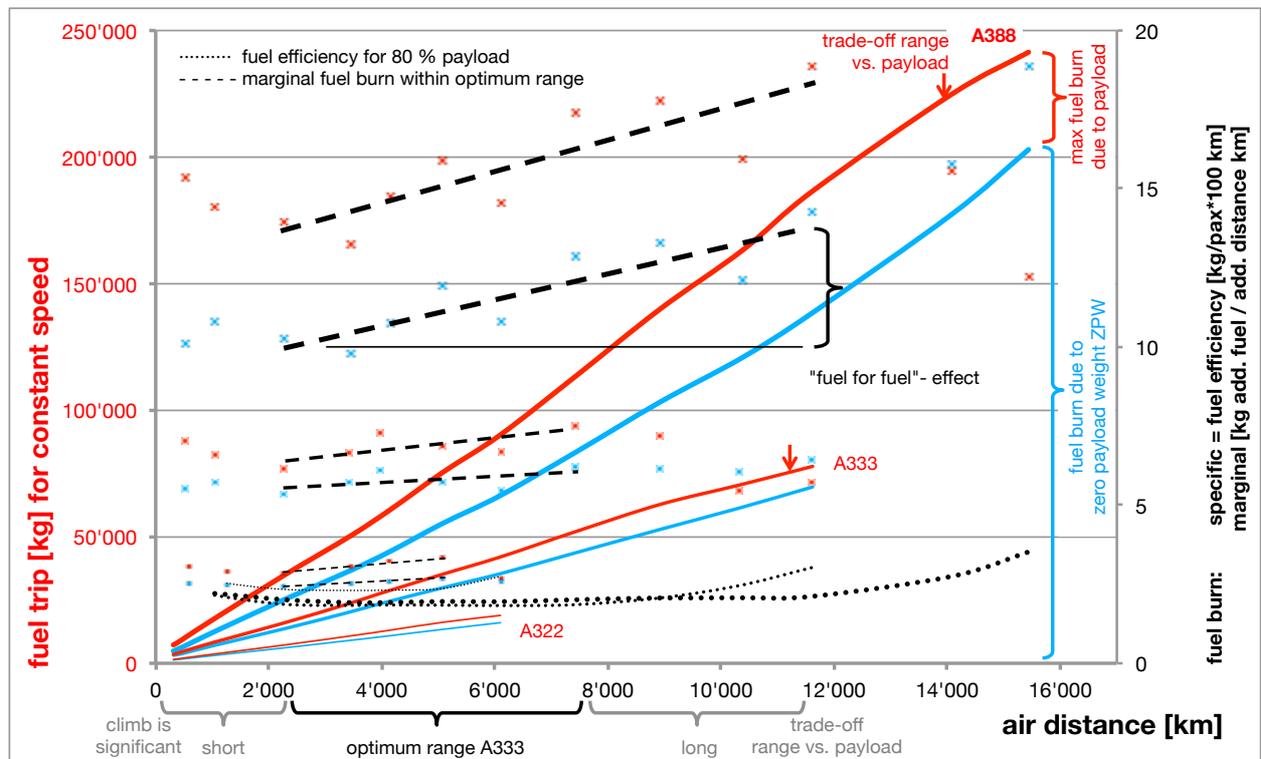


fig. 11 Impact of distance on fuel burn:

Fuel Burn (fuel trip) for zero to maximum payload) for distance 250 km to max. (left scale);

Specific Fuel Burn SFB [kg fuel/pax \* 100 km] for distance 250 km to max. (right scale)

Marginal Fuel Burn MFB [kg add. fuel/100 kg add. payload \* 1'000 km] for optimum range (r. scale)

Marginal fuel burn as a function of distance is approximately linear. Thus, fuel trip as a function of distance is slightly progressive. This effect can best be seen on the curve for marginal fuel burn and on the curves for fuel burn for A388 (fig. 11).

$$MFB_{dist.} = MFB_{d,2'000} + \frac{\Delta \text{fuel trip [kg]}}{\Delta \text{distance [km]}} \cdot \text{dist. [km]}$$

$$\text{gradient: } \frac{\Delta \text{fuel trip [kg]}}{\Delta \text{distance [km]}} = MFB_{dist. \text{ optimum range}}$$

Fuel burn increases slightly progressively due to the "fuel for fuel" effect: additional fuel has to be carried over a longer distance to be available at the end of the flight, which increases weight and therefore fuel burn on the distance to the point of use of this fuel.

$$\text{fuel trip [kg]} = ft_{2'000} + MFB_{distance} \cdot \text{dist. [km]}$$

Ratio of fuel burn for long over short distance is:

$$\frac{ft_{max}}{ft_{500}} \sim 10 \text{ to } 34 \text{ for distances } > 500 \text{ km.}$$

The set of parameters can be used within optimum range (~ 2'000 to 5'000 / 7'000 / 12'000 km):

air-craft	$ft_{2'000}$ [kto]	$MFB_{d,2'000}$ [ $\frac{kg \text{ fuel}}{2'000 \text{ km}}$ ]	$\frac{\Delta ft}{\Delta d}$ [ $\frac{kg}{km}$ ]	$MFB_{d,max}$ [ $\frac{kg \text{ fuel}}{2'000 \text{ km}}$ ]	$ft_{max}$ [kto]	$\frac{ft_{p,max}}{ft_{p,2'000}}$ [ratio]
A322	6.1	2.4	$10 \cdot 10^{-5}$	2.7	13	2.2
	7.2	2.9	$16 \cdot 10^{-5}$	3.3	16	2.3
A333	14	5.3	$17 \cdot 10^{-5}$	6.2	43	3.2
	16	6.2	$26 \cdot 10^{-5}$	7.5	52	3.3
A388	25	10	$43 \cdot 10^{-5}$	14	178	7.1
	35	14	$53 \cdot 10^{-5}$	19	225	6.5

fig. 13 Set of parameters for fuel burn as function of distance within optimum range for  
- zero payload (first row),  
- max. payload (second row).

For a short-distance flight, the impact of the climb profile and, of course, greater fuel burn per distance would have to be taken into account. For a long-distance flight, the effect of the trade-off between distance and payload would be significant for marginal fuel burn (no further discussion in this paper).

## Impact of distance on CASK

The impact of distance on CASK is very similar for all aircraft types as a result of the greater number of seats available and higher marginal fuel burn per aircraft.

air-craft	occ. seats [number]	MFB [kg/km]	$\Delta \text{cost of fuel}$ [ $\frac{\text{€cent}}{\text{km}}$ ]	$\Delta \text{CASK}_d$ [ $\frac{\text{€cent}}{\text{km}}$ ]
A322	0	2.2 - 2.7	$10 \cdot 10^{-3}$	
	120		$15 \cdot 10^{-3}$	$1.3 \cdot 10^{-5}$
	150	2.9 - 3.3	$16 \cdot 10^{-3}$	
A333	0	5.3 - 6.2	$17 \cdot 10^{-3}$	
	216		$24 \cdot 10^{-3}$	$1.1 \cdot 10^{-5}$
	277	6.2 - 7.5	$26 \cdot 10^{-3}$	
A388	0	10 - 14	$43 \cdot 10^{-3}$	
	435		$51 \cdot 10^{-3}$	$1.2 \cdot 10^{-5}$
	544	14 - 19	$53 \cdot 10^{-3}$	

fig. 14: Impact of distance on  $\text{CASK}_d$  for  
- zero payload (first row),  
- average occupancy rate (~ 80%),  
- max. payload (third row).

## 5. Impact of weight and distance

The impact of weight and distance on fuel burn (fuel trip) can be described as one function collectively:

$$\begin{aligned} ft \text{ [kg]} &= ft_{p,2'000} + MFB_p \cdot p + MFB_d \cdot d \\ &= ft_{d,2'000} \\ &+ \left( MFB_{p,2'000} + \frac{\Delta ft}{\Delta p} \cdot d \right) \cdot p \\ &+ \left( MFB_{d,2'000} + \frac{\Delta ft}{\Delta d} \cdot d \right) \cdot d \end{aligned}$$

air-craft	$ft_{2'000}$ [kto]	$MFB_{2'000}$ [ $\frac{kg f}{kg p}$ ]	$\frac{\Delta ft}{\Delta p}$ [ $\frac{kg}{kg \cdot km}$ ]	$MFB_{2'000}$ [ $\frac{kg f}{2'000 \text{ km}}$ ]	$\frac{\Delta ft}{\Delta d}$ [ $\frac{kg}{km}$ ]	$ft_{max}$ [kto]
A322	6.1	0.07	$3.5 \cdot 10^{-5}$	2.4	$10 \cdot 10^{-5}$	16
A333	14	0.05	$2.5 \cdot 10^{-5}$	5.3	$17 \cdot 10^{-5}$	52
A388	25	0.10	$4.4 \cdot 10^{-5}$	10	$43 \cdot 10^{-5}$	225

$ft$  = fuel trip  $p$  = payload [kg]  $d$  = distance [km]

fig. 15 Set of parameters for fuel burn as a function of distance and payload within opt. range.

This set of parameters for calculation of fuel trip can be displayed as a function of payload the same way as in fig. 7a) and fig. 7b) accordingly for flights of a given distance. The trade-off between distance and payload can be seen in the upper right-hand corner for each type of aircraft (fig. 16).

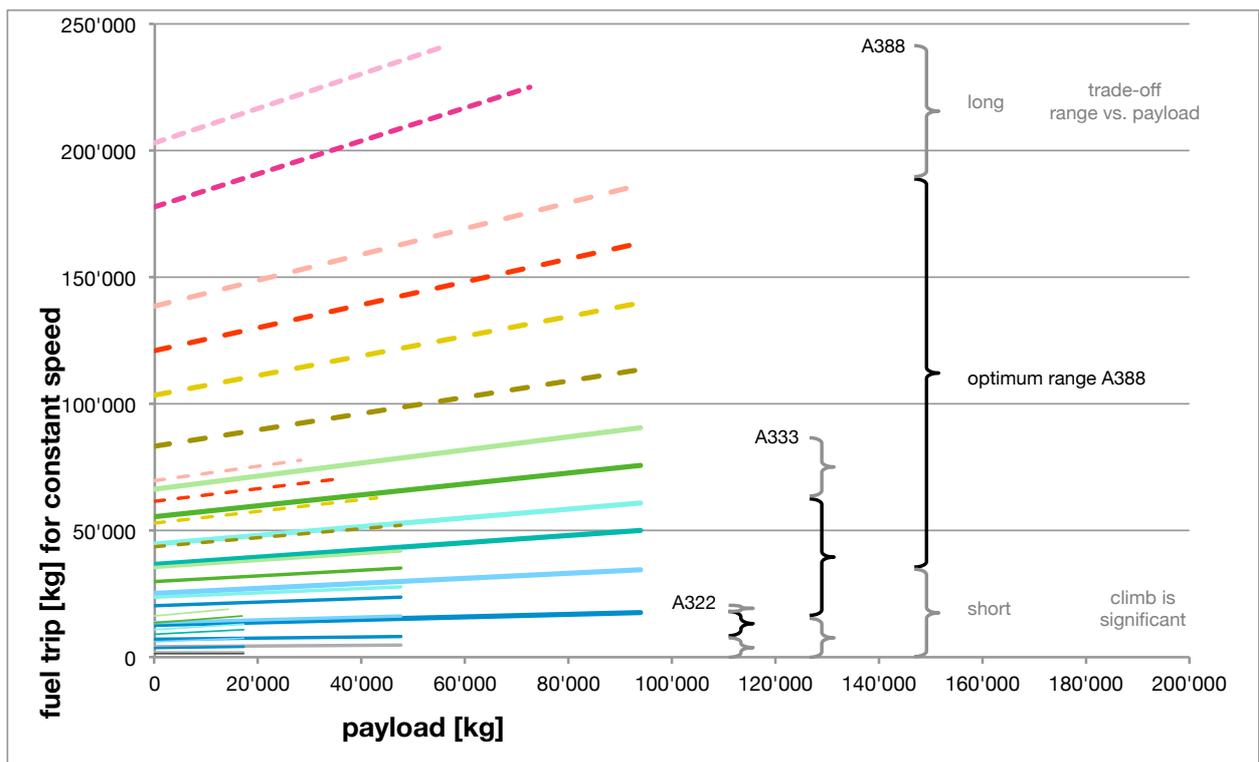
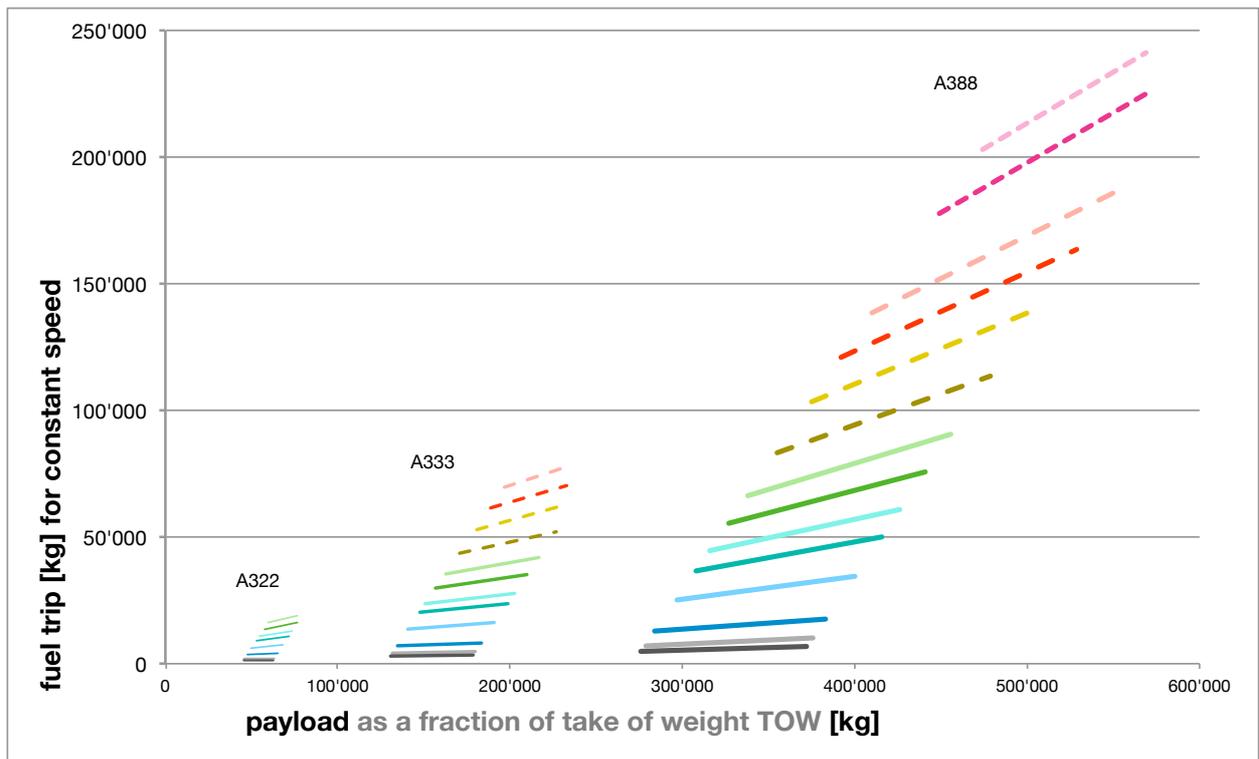
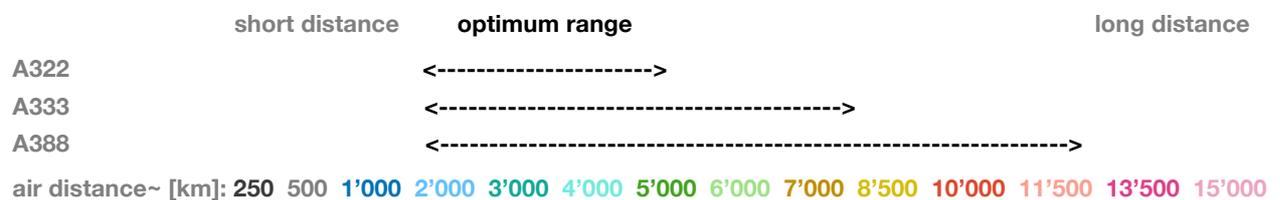


fig. 16 Fuel trip [kg] as function of payload [kg] for distances 1'000 - 15'000 km and constant true air speed  
 a) Payload [kg] as a fraction of takeoff weight, thus fuel trip [kg] for zero to maximum payload.  
 b) Fuel trip [kg] as function of payload [kg].



## 6. Impact of optimised cruise speed

### Range performance

Constant operating speed is a useful assumption to understand the impact of weight and distance on fuel burn, but it is not the most efficient method for operating an aircraft.

Efficient range operation of an airplane can either mean a) extract the maximum flying distance from a given fuel load or b) to fly a specified distance with minimum expenditure of fuel, represented by

$$\text{specific range} = \frac{\text{distance [km]}}{\text{fuel burn [kg]}} = \frac{\text{velocity} \left[ \frac{\text{km}}{\text{h}} \right]}{\text{fuel flow} \left[ \frac{\text{kg}}{\text{h}} \right]}$$

If maximum specific range is desired, the flight condition must provide a maximum of velocity fuel flow and therefore minimal drag. This particular point would be located by drawing a straight line from the origin tangent to the curve of fuel flow versus velocity [7] (fig. 13a).

Further insights could be gained by discussing max. endurance (equal minimal drag condition =  $L/D_{max}$ ).

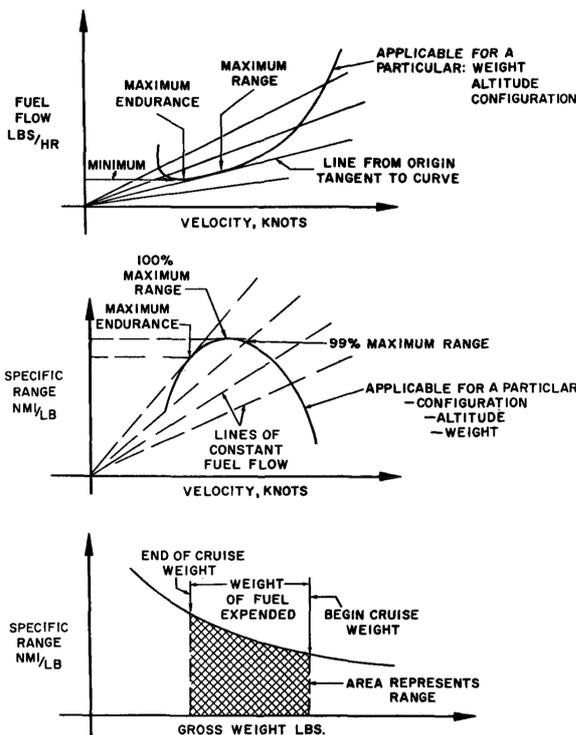


fig. 13a-c, from [7]: fuel flow vs. velocity  
 specific range vs. velocity  
 specific range vs. gross weight

This means that maximum range [ ] is achieved at one (specific) a.o.a. The tangent on the drag curve [ ] corresponds to the a.o.a. that produces the mathematical relationship of  $\sqrt{L/D_{max}}$ . In subsonic

performance,  $\sqrt{L/D_{max}}$  occurs at one particular value of a.o.a. and like its relative to  $L/D_{max}$  is unaffected by weight or altitude.

MRC is always achieved at the highest altitude at which  $\sqrt{L/D_{max}}$  can be sustained with maximum cruise power. [9]

### Control systems engineering (cybernetics)

The point of maximum endurance divides the chart into two sections of different systemic behaviour:

- for  $v < \sqrt{L/D_{max}}$  a reduction of speed is reinforced by increasing drag, an increase of speed is reinforced by decreasing drag;
- for  $v > \sqrt{L/D_{max}}$  every speed change is damped by the corresponding drag change: reduced speed means reduced drag, increased speed means increased drag as well.

The steeper the curve is (the more on the left or right of the divide), the greater the reinforcing or damping effect will be.

Thus, flying in the (right) sector where  $v > \sqrt{L/D_{max}}$  is more stable and oscillation is relatively small, while flying in the (left) sector where  $v < \sqrt{L/D_{max}}$  needs more and quicker response to control the aircraft by throttle (and trim), which becomes even more difficult with external parameters such as wind change. Maintaining the aircraft at maximum endurance speed would mean oscillating between the two sectors.

It is therefore reasonable to define a point on the velocity-fuel flow curve in the right sector not too close to  $\sqrt{L/D_{max}}$ . This point can be obtained by one specific a.o.a. Knowing this angle and being able accurately to read it in flight, it is possible to "fly a.o.a.", which means to establish a specific range cruise (with the desired economical cruising or holding conditions) in a simple way - with reference to just one instrument and without any Mach schedules or complex charts with variables, e.g. temperature, weight, altitude, power setting or speed calculations and therefore avoiding inaccuracies. Flying a.o.a. can also be an excellent cross-check of all other performance indicators. [9] Further insights could be gained by discussing these issues for climb and descent.

### Velocity as function of fuel burn

To obtain this one specific a.o.a. at a given altitude, while losing weight due to fuel burn, the only way is to reduce velocity. Since

$$a.o.a. = \frac{C_L}{0.11} = \frac{W \cdot 2 \cdot g}{\rho \cdot A \cdot v^2 \cdot 0.11} \quad \text{there is } \Delta v = \sqrt{\Delta W}$$

Velocity diminishes with the square root of reduction of weight. In other words: the lighter an aircraft gets,

the slower its optimum speed to maintain a.o.a. that minimizes induced drag ceteris paribus. That change in velocity can be remarkably large.

air-craft	fuel burn max. [to]	TOW max [to]	TAS [m/s]	$\Delta W_c$ [%]	$\Delta TAS$ [%]	$\Delta TAS$ [m/s]
A322	~ 20	~ 77	~ 230	- 26	- 14	- 32
A333	~ 76	~ 233	~ 242	- 33	- 18	- 43
A388	~ 241	~ 569	~ 251	- 42	- 24	- 60

fig. 14 Variation of true air speed TAS for max. to min. fuel load for cruising long distances.

As an alternative to diminishing velocity, altitude can be increased. Further insights could be gained by discussing typical flight profiles for velocity and altitude.

### Long-range cruise: LRC

Historically, LRC has been defined as the speed above MRC that will result in a 1 percent decrease in fuel mileage [10]. Most long-range cruise operation is conducted at the flight condition which provides 99 % of the absolute maximum specific range. The advantage is that 1 percent of range is traded for ~ 3 to 5 % higher cruise velocity. Since higher cruise speed has a great number of advantages, the small sacrifice of range is a fair bargain. [7] Modern flight management systems automatically adjust LRC speed throughout cruise for weight change due to fuel burn, (as well as changes in cruise altitude). [10]

### Minimized operating cost: ECON

Speed selection is crucial to fuel burn and trip time (for a given cruising altitude).

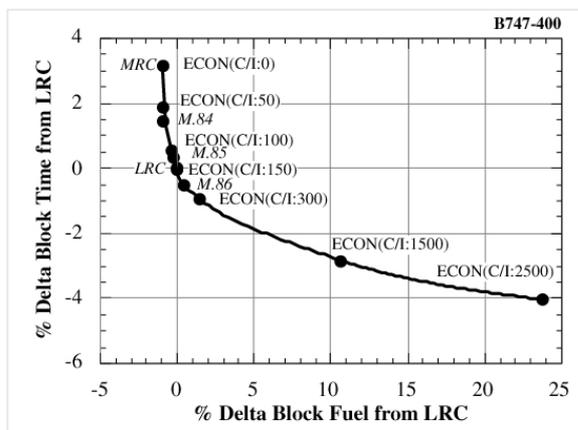


fig. 15 Trade-off between time and fuel usage: MRC, LRC and ECON for CI 0 to 2500. (from [11])

To minimize operating costs, both optimum fuel mileage (either maximum range for a given amount of fuel or minimum fuel usage for a given distance) and time costs (labour, aircraft etc.) have to be taken into account. ECON speed is therefore calculated as a function of

$$\text{cost index } CI = \frac{\text{Time Cost } \left[ \frac{\$}{\text{hr}} \right]}{\text{Fuel Cost } \left[ \frac{\text{cents}}{\text{lb}} \right]}$$

and controlled and adjusted throughout cruise.

Neglecting time cost, fuel is minimized, this means the aircraft is operating at maximum-range cruise (MRC), while the traditional speed is long-range cruise (LRC, see above).

### Empirical analysis of OFF<sub>sim</sub>

Empirical analysis for operations conducted with LRC-scheme shows longer flight times and some savings in fuel compared to operations conducted with constant speed — as would be expected from theoretical concepts. Differences are not very great, however, and basic relationships/functions stay the same.

## 7. Conclusion and findings

### Overview

Empirical analysis of operations using OFF<sub>sim</sub> allows us to outline basic key figures for the relation of main parameters of aircraft type, or trip distance and fuel burn. This can be useful for estimating the prospects of change in operational plans or structural issues for aircraft, hence innovations to reduce its weight. In this case, the fuel savings will be on every flight during the lifetime of the aircraft.

One may argue, that 4.0 kg of fuel burnt to transport 1 kg of payload over 15'000 km is a large number (e.g. Lisboa - Perth) – well, this is both true and not.

It is true because 15'000 km is outside of optimum range for any commercial aircraft. Optimum range is no more than ~ 12'000 km for an A388. Within this range, it takes ~ 0.2 kg fuel/kg payload \* 1'000 km - and if 15'000 km were within optimum range, the amount of fuel would only be ~ 2.6 kg fuel/kg payload instead of 4.0 kg/kg.

It is not true because within optimum range, fuel burnt to transport 1 kg of payload is about ~ 0.2 kg fuel/kg payload \* 1'000 km for commercial aircraft types studied. This figure is comparable to individual passenger land transportation.

## Impact of weight

A reduction in weight (structural or operating) by 1 kg results in a fuel saving of  $\sim 2$  to  $3 \cdot 10^{-5}$  [kg/km] = 0.02 to 0.03 [kg fuel / 1'000 km] respectively:

air-craft	distance [km]				
	2'000	4'000	5'000	7'000	12'000
A322	0.07	0.12	0.16	MFB [kg fuel/kg payl.]	
$\Delta$ cost of fuel	2.9	3.1	3.2	$\left[ \frac{\text{€cent}}{\text{kg payload} \cdot \text{km}} \right] \cdot 10^{-3}$	
$\Delta$ CASK <sub>p</sub>	3.2	3.4	3.6	$\left[ \frac{\text{‰}}{\text{kg payload}} \right] \cdot 10^{-3}$	
A333	0.05	0.08	0.11	0.18	
	2.1	2.1	2.2	2.4	
	<b>1.3</b>	<b>1.3</b>	<b>1.3</b>	<b>1.4</b>	
A388	0.10	0.17	0.18	0.26	0.45
	2.4	3.5	3.5	3.5	3.9
	<b>0.7</b>	<b>1.1</b>	<b>1.1</b>	<b>1.1</b>	<b>1.2</b>

ft = fuel trip payl. = payload [kg] d = distance [km]

fig. 15 Impact of weight for selected distances [km] on marginal fuel burn MFB (first row), on cost of fuel for 1€/kg fuel (second row), and on CASK<sub>p</sub> (third row).

To reduce CASK of 6 €cent by 1 ‰, weight of aircraft types studied must be reduced by  $\sim 300$  to 1'000 kg.

## Impact of distance

Optimum range is not only limited by a maximum distance of  $\sim 5'000$  to 12'000 km, but also by a minimal distance of  $\sim 2'000$  km.

For shorter distances, a significant portion of trip fuel is burnt for climb and therefore specific fuel burn is higher, thus operation is less efficient.

For longer distances, structural limits are the reason for trade-offs between fuel, payload and range. This makes operation less efficient as well.

The "Fuel for fuel" effect is verified by empirical analysis. As it turns out, it is relatively small compared to the effects of operating a flight outside of optimum range.

## Optimised cruise speed

Long-range cruise (LRC) operation puts the aircraft in an easy-to-control-state as well as providing greater economy compared to maximum-range cruise (MRC), taking into account the cost of time: velocity increases by  $\sim 3$  to 5 % while the absolute maximum specific range decreases by only 1 %.

Longer flight times and fuel savings due to LRC-scheme instead of constant speed can be verified by empirical analysis - as would be expected from theo-

retical concepts. Differences are not very great, however, and basic relationships/functions stay the same.

## 8. References and acknowledgements

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Thanks for the reviewing of this document to Manfrani, Leonardo Dr. Prof. and Neining, Bruno Dr. Prof. em. (both from ZHAW Zurich University of Applied Sciences, SoE School of Engineering, Winterthur).