

# Improvement of maintenance timetable stability based on iteratively assigning event flexibility in FPESP

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## Abstract

In the operational management of railway networks, the fast adaptation of timetable scenarios is an important requirement, in which operational disruptions or time windows with temporary unavailability of infrastructure, for instance during maintenance time windows, are taken into consideration. In those situations, easy and fast reconfiguration and recalculation of timetable data is of central importance. This local and temporal rescheduling results in shifted departure and arrival times and sometimes even in modified stop patterns at intermediate stations of train runs. In order to generate reliable timetabling results it is a prerequisite that train-track assignments as well as operational and commercial dependencies are taken into consideration. Therefore, it is crucial for the computer-aided planning process to refer to the right level of detail for the modelling of the track infrastructure and train dynamics. In this article we present a generic model that we call Track-Choice FPESP (TCFPESP), as it implements suitable extensions of the established PESP-model. We show how the service intention (the timetable specification resulting from line planning) together with resource capacity information can be utilized to configure the TCFPESP model.

In addition, we can calculate quantitative performance measures for assessing timetable quality aspects. To achieve this, we make use of the max-plus algebra for evaluating timetable stability. By utilizing delay impact values resulting from max-plus algebraic performance analysis, we are thus able to iteratively distribute event flexibility in such a way that overall stability of the maintenance timetable is improved.

This approach supports the planner to generate integrated periodic timetable solutions in iterative development cycles.

## Keywords

Flexible PESP, Mesoscopic railway topology, Service Intention, Timetabling with track assignment, Timetable stability analysis

## 1 Introduction

### 1.1 Generating timetable scenarios for short term planning

In the operational management of railway networks, an important requirement is the fast adaptation of timetable scenarios, in which operational disruptions or time windows

with temporary unavailability of infrastructure, as for instance during maintenance time windows ('possessions', see RailNetEurope (2017)), have to be accounted for. In those situations, easy and fast reconfiguration and recalculation of timetable data is of central importance. This local and temporal rescheduling results in shifted departure and arrival times and sometimes even in modified stop patterns at intermediate stations of train runs. Only recently, van Aken et al. (2017a) presented a PESP based macroscopic model for solving train timetable adjustment problems (TTAP) under infrastructure maintenance possessions (2017a). They show, that by applying TTAP, they are able to adjust a given timetable to a specified set of station track and complete open-track possessions by train retiming, reordering, short-turning and cancellation. In van Aken et al. (2017b) they apply several network aggregation techniques to reduce the problem size and thus enable the model to solve large instances within short computation times with instances of the complete Dutch railway network.

However, in order to generate reliable timetabling results it is prerequisite that besides train-track assignments, also operational and commercial dependencies are taken into consideration. Hence, finding the right level of detail for modelling track infrastructure and train dynamics is crucial for supporting the planning process in an optimal way.

In day-to-day business, determining the feasible event times for individual train runs and the corresponding resource-allocation fitting into efficient transport chains resulting from an integrated clockface timetable is time-consuming and is carried out manually. On the other hand, algorithmic approaches for solving this task computationally require models based on microscopic information about track capacity. This capacity information is aggregated to (normative) minimum headway constraints that are used for solving standard periodic timetabling problems. To facilitate this step, several research groups made suggestions, how to combine common timetabling procedures with constraints resulting from mesoscopic infrastructure information. Hansen and Pachl (2008) show how running, dwell and headway times at critical route nodes and platform tracks must be taken into account for train processing and present a deep timetable quality analysis depending on these parameters. De Fabris et al. (2014) calculate arrival and departure times, platform and route assignments in stations and junctions that trains visit along their lines. Bešinović et al. (2016) present a micro–macro framework based on an integrated iterative approach for computing a microscopically conflict-free timetable that uses a macroscopic optimization model with a post-processing stability evaluation. Caimi et al. (2011) extend PESP (see e.g. Serafini and Ukovich (1989) and Liebchen and Möhring (2007)) and propose the flexible periodic event scheduling problem (FPESP), where intervals are generated instead of fixed event times. By applying FPESP, the output does not define a final timetable but an input for finding a feasible timetable on a microscopic level, (Caimi (2009) and Caimi et al. (2009)).

## **1.2 Service Intention based approach for timetable specification**

To improve customer value even under limited operating conditions, such as those encountered during infrastructure maintenance intervals, our modelling approach for creating temporary schedules is also based on an extension of PESP and takes the 'service intention' (SI) as input data. The SI was first described in Wüst et al. (2008), formally specified in Caimi (2009) and integrates commercial timetabling requirements given by the respective demand oriented 'line concept' on one side and technical constraints on the other. The 'line concept' results from a strategical planning process which is executed by the transport carrier. In this process, the available amount, the dynamics and the circulation of rolling stock are taken into account. In Switzerland, the integrated fixed-

interval timetable (IFIT) is created on the basis of SI's. The required system times (minimum travel times between node stations, see for example Herrigel (2015) ) are a prerequisite (see e.g. Liebchen and Möhring (2007)).

The maintenance interval planning step (denoted as IP in the sequel) is executed by the infrastructure manager. In this step, the functional requirements of the SI are brought together with this mesoscopic infrastructure data model of a given scenario. Altogether these data can be maintained in a standard timetable editor (see for instance SMA Viriato, 2018). In this way, the SI represents functional timetabling requirements including line data, line frequencies and separations as well as line transfers at specific stations. Hence, it contains explicit information about intended transport chains but is still flexible enough, to allow different ways of operational planning and resource allocation. Like de Fabris et al. (2014), we call this level of abstraction of the available resources 'mesoscopic topology'. We call our FPESP model that we apply to this mesoscopic topology 'Track-Choice FPESP' (TCFPESP). To facilitate the problem of searching feasible solutions for local resource restrictions during maintenance intervals we make the assumption, that the train network outside the maintenance corridor is not affected by the restrictions at the level of mesoscopic topology. This allows us to separate the network into aggregated network partitions outside the IP relevant corridor and the disaggregated network partition at mesoscopic topology level. This network segmentation has also some similarity to the decomposition approach suggested by Lamorgese et al. (2016). They present an iterative dispatching algorithm in which the network is sequentially decomposed into a macroscopic line dispatching (master) and a microscopic station dispatching (slave) algorithm.

To evaluate timetable stability criteria we use a special algebraic approach that is commonly known as max-plus algebra. This approach has been elaborated in mathematical detail by Goverde (2007) who also demonstrates the benefits of this algebraic approach for timetable stability analysis in practical applications. According to this approach, timetable stability is defined in terms of the difference between the timetable period  $T$  and  $\lambda_0$ , defined as the maximum cycle mean over all circuits in the event activity network. If  $\lambda_0 < T$  the timetable is considered to be stable. We apply this method for evaluating the stability of our resulting timetable and try to improve the timetable based on this performance evaluation in successive re-planning iterations. More specifically, we show how the max-plus-delay impact analysis can help to improve timetable stability by iteratively adjusting local flexibility constraints in the configuration of the TCFPESP model.

As we want to demonstrate the operational benefit that can be obtained by utilizing the max-plus stability analysis for TCFPESP based re-planning, we finally present a case study without fixed time constraints for the planning step. This configuration represents the use case of designing a new timetable rather than the use case for altering an existing timetable, which is the typical constellation when planning temporary timetables for maintenance intervals. However, we think that in this way we can clearly point out the mentioned relationship between the planning step and the performance analysis step.

### 1.3 Structure of this article

This article is structured as follows: In section 2, we describe the methodology for achieving the research goals. In section 2.1 we summarize the FPESP model which implements the idea of periodic timetabling with event flexibility. Extending this FPESP to our proposed mesoscopic model we present in Section 2.2 our TCFPESP-model. For the iterative configuration of the event flexibility in the TCFPESP we make use of the

delay impact vector that we obtain from max-plus analysis. This is shown in section 2.3. In section 2.4 we describe the TCFPESP heuristic for reducing the overall delay impact. In section 3 (Case Study ‘Kerenzerberg’) we present the results from applying the methods introduced in section 2 and the coordinated application in a real-world scenario from eastern Switzerland. Finally, in section 4 we conclude with a summary of the findings and an outlook on future work.

## 2 Methodology

### 2.1 Periodic Timetabling with Event Flexibility

The classical PESP tries to determine a periodic schedule on the macroscopic level (i.e. without using the tracks at an operation point) within a period  $T$ . Event  $e \in E$  takes place at time  $\pi_e \in [0, T)$ . The schedule is periodic with time period  $T$ , hence each event is repeated periodically  $\{\dots, \pi_e - T, \pi_e, \pi_e + T, \pi_e + 2T, \dots\}$ .

The choices of the event times  $\pi_e$  depend on each other. The dependencies are described by arcs  $a = (e, f)$  from a set  $A$  and modelled as constraints in the PESP. The constraints always concern the two events  $e$  and  $f$  and define the minimum and maximum periodic time difference  $l_a$  and  $u_a$  between them. These bounds are given as parameters in the PESP model. We therefore look for the event times  $\pi_e$  for every  $e \in E$  that fulfill all constraints of the form

$$l_a \leq \pi_f - \pi_e + p_a T \leq u_a$$

for all  $a = (e, f) \in A$ , where  $p_a$  is an integer variable that makes sure, that these constraints are met in a periodic sense.

In order to avoid tedious iterations between the process steps “microscopic capacity planning” and “mesoscopic capacity planning” in case of infeasibility of the micro-level problem, one can improve the chance of finding a feasible solution by enlarging the solution space in the micro-level. This approach has been described in detail in Caimi et al. (2011b). We also implement this event flexibility method by adding some flexibility for the events of the event and activity network  $(E, A)$  by introducing lower and upper bounds to the event times of the arrival and departure nodes in Figure 1b. The final choice of the event times in the range between the lower and upper bound shall be independent for each event such that each value of the end of an activity arc should be reachable from each time value at beginning of that activity arc (see Figure 1a).

We are not forced to add this flexibility to all the events, but we can select the nodes where we want to add it, for instance only nodes corresponding to events in a main station area with high traffic density, where it is more difficult to schedule trains on the microscopic level. In general, one can say, that this placement of flexibility is the timetable configuration feature, which has the highest level of influence on improving operational stability. This is where the information provided by the max-plus measures of delay impact (see section 2.3 et seq.) can be utilized in order to achieve timetable stability. For more details regarding the FPESP method, we refer to the article of Caimi et al. (2011b).

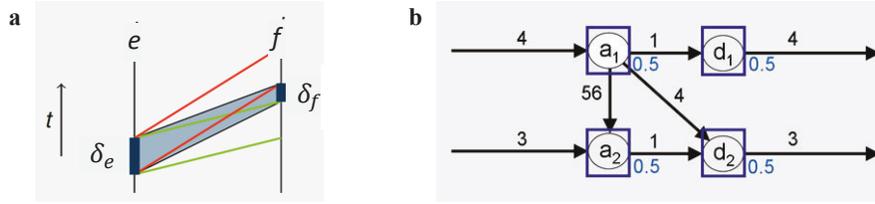


Figure 1: Target oriented placement of time reserves. a) Time frames  $[\pi_e, \pi_e + \delta_e]$  in place of time points  $\pi_e$ . By implementing this method, the normal PESP constraints  $l_a \leq \pi_f - \pi_e + p_a T \leq u_a$  now become  $l_a + \delta_e \leq \pi_f - \pi_e + p_a T \leq u_a - \delta_f$  (see next section). In the example b) this means that instead of planning time points  $(\pi_{a_1}, \pi_{d_1}, \pi_{a_2}, \pi_{d_2})$  we plan time frames  $[\pi_e, \pi_e + 0.5]$  for  $e \in \{a_1, d_1, a_2, d_2\}$ .

## 2.2 Track-choice FPESP.

For our proposed timetabling model, we extend the FPESP method with events at track-level in order to generate event slot timetables on a mesoscopic level. In the TCFPESP model, the mesoscopic infrastructure consisting of sections is summarized as a set  $I$  of operation points. Operation points are largely tracks and stations but can also be other critical resources such as junctions (see OP ‘Tiefenwinkel’ in Figure 2b). As mentioned before, each operation point  $i \in I$  is associated to a capacity consisting of a set of tracks  $T_i$ . A train run  $l \in L$  is described by a sequence of operation points of  $I$ .

Based on this mesoscopic model we form an event-activity network  $(E, A)$ . The set  $E$  of events consists of an arrival event  $arr_{li}$  and a departure event  $dep_{li}$  for each train run  $l \in L$  and operation point  $i \in I$ . The activities  $a \in A$  are directed arcs from  $E \times E$  and describe the dependencies between the events. For every train run we have arcs between arrival and departure events at the same operation points (dwell times or trip times) and arcs between departure and arrival events of successive operation points (running time between operation points). Further arcs include connections between train runs, headways and turnaround operations. Headway arcs  $a \in A_H$  are especially important for explaining the ‘track-choice FPESP model’ below. Headways are used to model safety distances between trains running in the same and in opposite directions. For the sake of simplicity in the formal description of the TCFPESP we consider only headways related to one operation point, i.e. we omit headways for train runs in opposite directions over several successive operation points. The TCFPESP-model can be easily extended to include general headways. They are included in our implementation of the timetable model.

We extended the classical PESP resp. FPESP model by using the number of tracks  $T_i$  at each operation point  $i \in I$ . The track-choice FPESP model assigns the arrival event  $arr_{li}$  and the departure event  $dep_{li}$  of train run  $l$  at operation point  $i$  uniquely to a track in  $T_i$ . We can use these assignments to switch on headway arcs  $a \in A_H$  by using the following big-M-approach. In addition to variables  $\pi$  and  $p$  from the classical PESP model we need:

- (i) Binary variables  $tc_{et}$  (track choice) for each event  $e \in E$  and track  $t \in T_{i(e)}$ , where operation point  $i(e)$  is associated to event  $e$ , i.e.  $e$  is equal to  $arr_{li}$  or  $dep_{li}$  for a train run  $l$ .
- (ii) Binary variables  $h_a$  for every headway edge  $a = (e, f) \in A_H$ . As mentioned before, headway edges are always between events at the same operation point, therefore  $T_{i(e)} = T_{i(f)}$  holds.

(iii) Positive variables  $\delta_e$  for each event  $e \in E$  to model the event flexibility. The track-choice model is defined by:

min  $f(\pi, \delta)$

$$s.t. \quad l_a + \delta_e \leq \pi_f - \pi_e + p_a T \leq u_a - \delta_f, \quad \forall a = (e, f) \in A \setminus A_H, \quad (1)$$

$$l_a + \delta_e - (1 - h_a)M \leq \pi_f - \pi_e + p_a T \leq u_a - \delta_f + (1 - h_a)M, \quad \forall a = (e, f) \in A_H, \quad (2)$$

$$\sum_{t \in T_i(e)} tc_{et} = 1, \quad \forall e \in E, \quad (3)$$

$$tc_{arr_{it}} = tc_{dep_{it}}, \quad \forall l \in L, i \in l, t \in T_i, \quad (4)$$

$$h_a \geq tc_{et} + tc_{ft} - 1, \quad \forall a = (e, f) \in A_H, \quad (5)$$

$$tc_{et}, h_a \in \{0, 1\}, \pi_e \in [0, T], p_a \in \mathbb{Z}, \delta_e \geq 0, \quad \forall e \in E, t \in T_i(e), a \in A,$$

where  $M$  is a big enough natural number.

In (1) the normal FPESP constraints are summarized (without headway arcs). In (2) are the headway constraints, which can be switched off with a big-M technique. The assignment of the events to the tracks is done in (3). (4) is used to assign the corresponding arrival and departure events to the same track. In (5) the headway variable is set to 1, if the events take place on the same track, i.e. the headway is required at this operation point.

There are many different objective functions  $f(\pi, \delta)$  suggested by Caimi et al. (2011b) for the FPESP model. To generate the traffic plan for our test scenario we use iteratively the TCFPESP with different objective functions (see Wüst et al. (2018b)), namely:

- We minimize all passenger relevant times (i.e.  $t \in A_T$  the set of trip arcs,  $d \in A_D$  the set of dwell arcs and  $c \in A_C$  the set of connections times). The weights  $w_t, w_d$  and  $w_c$  can be used for prioritizing certain times, e.g. connection times. We will call the model in this case MINTRAVEL, according to Caimi et al. (2011b). The objective function is defined as follows:

$$f_{TT}(\pi) = \sum_{t \in A_T} w_t \pi_t + \sum_{d \in A_D} w_d \pi_d + \sum_{c \in A_C} w_c \pi_c \quad (6)$$

- We maximize the flexibility in a certain range at certain arrival and departure events. The objective function is defined as follows:

$$f_{flex}(\delta) = \sum_{e \in V} w_e \delta_e, \quad (7)$$

where  $V \subseteq E$  is the set of all events where flexibility is introduced.

Furthermore we add two constraints. The passenger travel time  $f_{TT}$  has to be smaller than  $(1 + \epsilon)$  times the best possible travel time  $f_{TT}^*$  from the model MINTRAVEL. The flexibility for all events is bounded by a maximal flexibility  $\delta_{max}$  for a better distribution of the flexibility to all events. The two constraints are given by

$$f_{TT}(\pi) \leq (1 + \epsilon)f_{TT}^* \quad \text{and} \quad \delta_e \leq \delta_{max} \quad \forall e \in E, \quad (8)$$

where  $f_{TT}^*$  is the optimal value found for  $f_{TT}$  in (6).

We will call the model in this case CONTRAVEL according to Caimi et al. (2011b).  $\epsilon$  is a parameter controlling the quality of the schedule for the passengers' travel times and the weights  $w_e$  will be used for individual adjustments in event flexibility in order to maximize timetable stability (see section 2.3 and 2.4).

Both models MINTRAVEL and CONTRAVEL are therefore mixed integer linear problems. By using the models MINTRAVEL and CONTRAVEL iteratively we can generate a traffic plan covering stability and travelling time aspects (see Wüst et al. (2018b)).

### 2.3 Computation of the Cumulative Delay Impact

The Cumulative Delay Impact (CDI) is a measure to quantify the overall impact that a certain delay  $\kappa$  at a specific event  $f$  has on all other events  $e$ . Formally the CDI is computed as follows:

$$CDI_f(R) = \sum_{e \in E \setminus f} \max(\kappa - r_{ef}, 0)^\gamma, \quad (9)$$

where  $E$  denotes the set of all events.  $R$  represents the recovery matrix of size  $|E| \times |E|$  and  $r_{ef}$  represents the actual buffer time between events  $f$  and  $e$  given a periodic timetable  $\pi$  (For the details on the calculation of the recovery matrix  $R$  and the buffer times  $r_{ef}$  see Goverde (2005, 2007)). The event times are resulting from TCFPESP by taking the lower bounds of the event time intervals calculated.

$\kappa$  is the parameter that denotes the initial delay (in minutes) applied to node  $f$ , for which  $CDI_f$  shall be calculated. Finally,  $\gamma \geq 1$  is a parameter to increase the impact of positive differences between the delay  $\kappa$  and  $r_{ef}$ . In this study  $\gamma$  was always set to 1. Furthermore,  $CDI_f$  is strictly monotonically increasing in  $\kappa$  and  $CDI_f(R) = 0$  for  $\kappa = 0$ ,  $\gamma \geq 1$ . The initial delay  $\kappa$  can of course be set for each event  $f \in E$  individually, e.g. when  $\kappa$  is determined with the help of a statistical delay analysis for each event  $f \in E$ .

### 2.4 Heuristic for improvement of delay impact

We measure the stability of a periodic timetable  $\pi$  by the sum of all cumulative delay impacts, i.e. we consider  $f_{sta}(\pi) = \sum_{f \in E} CDI_f(R)$ . Given an acceptable  $\kappa$  (from an operational point of view), we would like to have this measure as small as possible. From the definition of CDI, it follows, that  $f_{sta}(\pi)$  is bounded from below by 0.

It would therefore be natural to use  $f_{sta}(\pi)$  in the CONTRAVEL model as objective function. Since we don't have a direct solution approach for this case, we propose the following heuristic.

Iteratively we try to use the weights  $w_f$  in the function  $f_{flex}(\delta)$  to give more flexibility to the events  $f \in E$ , where  $CDI_f(R) > 0$ . Weight  $w_f$  is computed as follows:

$$w_f = \begin{cases} \left( \frac{CDI_f(R)}{\max_{f \in E} CDI_f(R)} \right)^\alpha & \text{if } \max_{f \in E} CDI_f(R) > 0 \text{ and } CDI_f(R) \geq \theta \cdot CDI_{max} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

where  $CDI_{max} = \max_{f \in E} CDI_f(R)$  is the maximum CDI value observed,  $\theta$  is a

threshold parameter to determine, which values of  $CDI_f(R)$  are considered for subsequent weightings,  $\alpha \geq 1$  is a parameter to over proportionally increase the weights, the larger  $CDI_f(R)$  is.

**Iteration scheme: Improving delay impact**

**Input:**

- Periodic timetable  $\pi$  computed with the CONTRAVEL model. The weights  $w_f$  in the objective function are set to 1 for this initial timetable  $\pi$ .
- Set initial delay  $\kappa$  and parameter  $\alpha$  and  $\gamma$ .

**Iteration steps:**

**Step 1:** Compute  $f_{sta}(\pi)$  by summing up the  $CDI_f(R)$  for all  $f \in E$ .

**If**  $f_{sta}(\pi) = 0$ ,  
 Stop iteration and accept timetable  $\pi$ .

**Else**  
 Set  $\beta = f_{sta}(\pi)$ .

**End If**

**Step 2: While**  $f_{sta}(\pi) \leq \beta$

For timetable  $\pi$  set the weights  $w_f$  according to equation (10).

Recompute a new timetable  $\pi_{new}$  with the help of the CONTRAVEL model and the new weights  $w_f$ .

Compute  $f_{sta}(\pi_{new})$ .

**If**  $f_{sta}(\pi_{new}) < \beta$ ,  
 Set  $\beta = f_{sta}(\pi_{new})$  and  $\pi = \pi_{new}$ .

**Else**  
 Set  $\beta = (-1)$  (leave while loop).

**End if**

**End While**

**Step 3:** Accept timetable  $\pi$ .

In the iteration scheme above we compute in step 1 the sum of the cumulative delays of the initial timetable  $\pi$ . As mentioned above the timetable  $\pi$  from the CONTRAVEL model corresponds to computed lower bounds of the single events. In step 2 we enter a while loop as long as the adaption of the weights  $w_f$  leads to an improvement of the stability measure  $f_{sta}$ . The timetable  $\pi$  with the minimal stability measure  $f_{sta}$  during the iterations will be accepted at the end.

All timetables during the iterations fulfil the same service intention (see section 1.2), but the resulting timetable is the most robust one with respect to the cumulative delay impact measure (among the constructed timetables during iterations). We illustrate this iteration scheme in our case study in section 3.

### 3 Case study ‘Kerenzerberg’

In order to illustrate the iterative improvement of timetable stability for IP scenarios, we selected a railway corridor in the eastern part of Switzerland. We call the case study ‘Kerenzerberg’ and the maintenance work is planned on the network section between Flums and Mels. The impact on the schedule is that there is a reduced velocity on that

section during normal operation hours.

### 3.1 Network segmentation

To avoid putting too much effort into entering information that is not needed and rather focus on the relevant perimeter for the IP timetabling scenario, one has to identify which part of the entire railway network has to be investigated and which part will be assumed to remain as given by the ordinary timetable. In a first step, the relevant lines and services operating on the subnetwork, which will be affected by the construction sites, have to be identified. In a second step, those lines, which are coupled (e.g. by transfers or technical dependencies) to these affected lines have to be found.

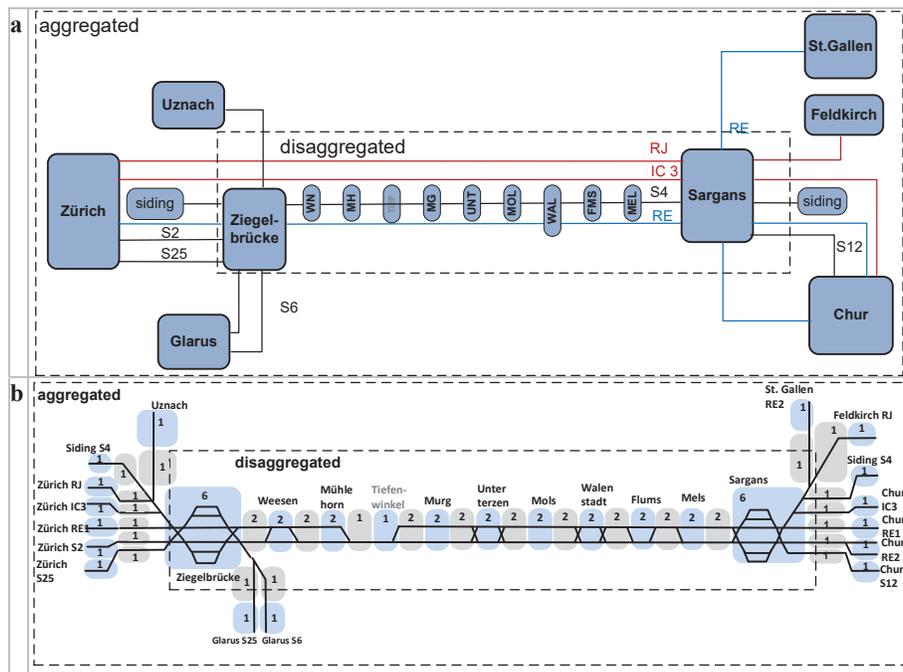


Figure 2: Case study Kerenzberg a) In order to divide the relevant infrastructure for the IP timetabling scenario into a network partition with the relevant level of detail and a peripheral part with more coarse information, the railway network is divided into subnetworks. A disaggregated subnetwork containing the relevant infrastructure segments at mesoscopic level and an aggregated subnetwork, representing simplified infrastructure on the macroscopic level. b) Shows the track topology for the both, the aggregated and disaggregated network partitions. The grey shaded topology points represent section type operation points, light blue shaded topology points indicate operation points. Numbers indicate the topology point's number of tracks. In order to avoid treating line interactions outside the disaggregated partition, each line has an individual periphery with a section between the final destination point and the boundary operation point. The section that separates the two partitions from each other is configured with aggregated running times and dwell times of the respective line.

One has to identify the sub-network nodes which isolate the relevant infrastructure partitions from the fixed periphery. In this way one obtains a disaggregated subnetwork containing the relevant infrastructure segments and an aggregated subnetwork, representing infrastructure on the macroscopic level (see outer dashed square areas in Figure 2a and b). The disaggregated subnetwork is configured with all mesoscopic details. On this disaggregated subnetwork all train movements are planned in detail. For each line coming from or going beyond the boundary nodes of the disaggregated subnetwork we create a virtual end station node which is connected by a single section to the corresponding boundary node. The section lengths with the appropriate trip times, the turnaround times of the line outside the disaggregated subnetwork together with the run- and dwell times within the disaggregated subnetwork have to sum up to the proper roundtrip time. The mesoscopic track topology of the disaggregated subnetwork is illustrated in Figure 2b).

### 3.2 Network of the case study Kerenzerberg.

The planned construction or maintenance work for our test scenario ‘Kerenzerberg’ is located on the network section between Flums and Mels. During the IP interval, trains are running with reduced speed in both directions. We decided to use the corridor Ziegelbrücke-Sargans as the disaggregated partition of the test network. It has to be mentioned, that there is a single-track section between the operation points ‘Mühlehorn’ and ‘Tiefenwinkel’. For this disaggregated network partition, we iteratively generate IP timetable scenarios (see section 3.4). The western part of Ziegelbrücke is aggregated, i.e. we introduced the nodes Uznach, Zürich, Glarus and a siding of Ziegelbrücke and connecting tracks. The aggregated network will be used to maintain vehicle circulation (e.g. turnarounds) aspects of lines and to model connections to peripheral lines (see the description of SI in section 3.3). The eastern part of Sargans also belongs to the aggregated partition. We introduced the nodes St.Gallen, Feldkirch, Chur and a siding of Sargans. In the aggregated network we assume to have enough track capacity to compensate for delays.

### 3.3 Description of Service Intention

The configuration of the SI is mainly done in the planning system Viriato. Additional information like upper boundaries of time intervals and flexibility of event times as required in the TCFPESP model is maintained in an R-based table editor (see chapter 2.2). The SI-lines represent the lines in the corresponding timetable 2018 with minor adaptations. In order to demonstrate the turnaround operations within our test scenario, we decided that the line S4 makes a turnaround in a siding next to Ziegelbrücke and Sargans, respectively. The other commuter lines (S x) rotate between a final station and a boundary node or between two final stations via a boundary node. Minimal line rotation times and line frequencies are indicated in Table 1.

Table 1: Line rotations and line frequencies

Line ID	Minimum line rotation time (min)	Line frequency (repetitions per hour)
S4	58.8	1
RJ	47.3	0.5

IC 3	43.8	1
RE 1	50.3	1
S12	12	1
S25	13	1
S6	16	1
RE 2	12	1
S 2	11	1

Table 1: Line rotations and frequencies. The minimum line rotation times are computed according to the approach of Liebchen and Möhring (2007). The corresponding turnaround intervals are computed in such a way, that a service with a minimal number of rolling stock is possible. In our case study the line S 4 is operating with one rolling stock. The other lines operate with more than one rolling stock due to longer round-trip times. These bounds are not computed according to Liebchen and Möhring (2007), they are set manually and have reduced line rotation times.

Ziegelbrücke and Sargans are considered as local hubs. At these stations the traffic plan has to account for passenger transfers between lines. Technically, these transfer requirements result in connections line constraints in our TCFPESP-model. These line connections are indicated in Table 2. For a detailed definition of the infrastructure and the SI specification including time intervals of running times, dwell times, turnaround times, separation times etc. see Wüst et al. (2018b).

Table 2: Line Connections at Stations

Connection [1, 15] From/To at station	S 25 (ZB-GL)	S 25 (GL-ZB)	S 4 (ZGB-SA)	S 4 (SA-ZGB)	S 12 (SA-CH)	RE 2 (CH-SG)	IC 3 (ZGB-SA)	RE 1 (SA-ZGB)	RE 1 (ZGB-SA)	S 6 (GL-UZ)	S 6 (UZ-GL)
S 4 (ZGB-SA)	ZGB	ZGB			SA						
S 4 (SA-ZGB)		ZGB			SA						
S 25 (GL-ZB)			ZGB								
S 25 (ZB-GL)				ZGB							
IC 3 (ZGB-SA)						SA					
S 12 (CH-SA)							SA				
RE 2 (CH-SG)								SA			
RE 2 (SG-CH)							SA	SA			
RE 1 (ZGB-SA)											ZGB
RE 1 (SA-ZGB)										ZGB	ZGB
S 6 (GL-UZ)								ZGB	ZGB		
S 6 (UZ-GL)								ZGB			

Table 2: Case study Kerenzerberg: Line connections at stations are dependent on the direction of the train runs. The time intervals for connection arcs [lb, ub] is configured identically for all connections: [1 min, 15 min].

### 3.4 Iterative improvement of timetable stability

Once the configuration of the SI and the mesoscopic infrastructure is complete it is

transformed into the TCFPESP model which was implemented in GAMS by applying the CONTRAVEL model as indicated by equations (7) and (8) with parameter  $\epsilon = 0.5$  and  $\delta_{max} = 10s$ . In case the SI is feasible with respect to the capacity constraints given by the infrastructure, GAMS returns the timetable  $\pi$  with flexibility  $\delta$ .

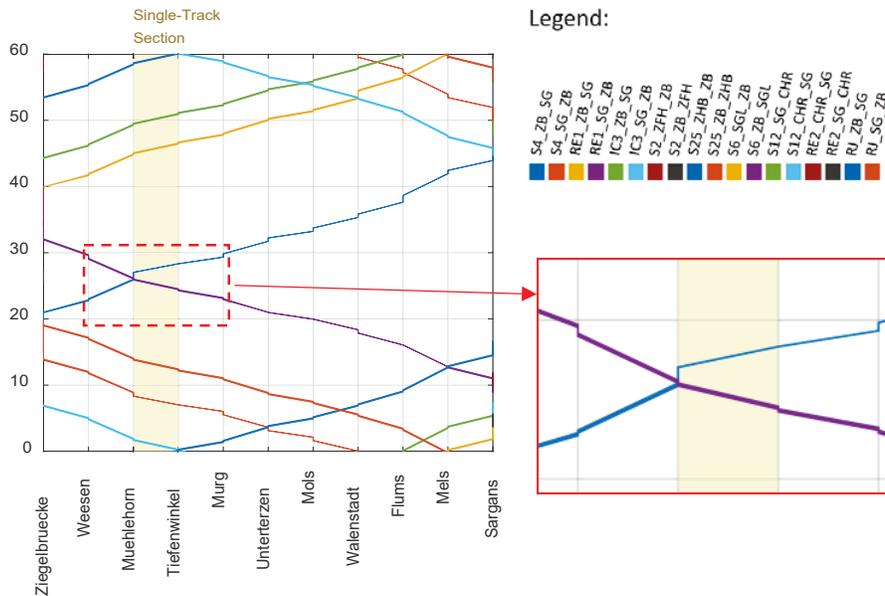


Figure 3: Time-distance diagram: GAMS output for TCFPESP applying step 1 of the iteration scheme of section 2.4. Line names and directions are indicated by colours as shown in the legend. One can also see the narrow but variable width of the capacity time bands indicating a low flexibility of each train run in a range below 10 seconds.

These are plotted as time-distance diagram as shown in Figure 3. This represents the result of the first step in the iteration scheme in section 2.4. The timetable is the result of a CONTRAVEL-model configuration (see equation (8) in section 2.4.). As can be seen in the diagram, the range of flexibility of the train runs is quite narrow which is due to a small  $\delta_{max} = 10s$ , but show variable width within a certain range up to  $\delta_{max}$ .

They are quite homogenously distributed, indicating some, but low flexibility in all timetable events. The stability of this result is assessed by calculating  $CDI_f(R)$  for an initial delay  $\kappa$  of 3 minutes. Figure 4 illustrates the delay impact of each timetable event to all other network events indicated by the corresponding colour (dark colours indicate higher delay impacts) together with the interdependencies (connecting arrows) in the event activity network. In order to demonstrate the influence a target oriented adjustment of the event flexibility, for step 2 of the iteration scheme (for details see section 2.4) we decided to define two rather different settings: (i) with a threshold of  $\theta = 0.95$  only a limited number of event nodes was selected for weighting, whereas (ii) with a threshold of  $\theta = 0.40$  quite a large number of event nodes was selected for weighting. The weights  $w_f$  were subsequently used to calculate a more robust timetable  $\pi$  (see equation 10). This

time the parameter  $\delta_{max}$  is set to 60 seconds in order to assign more flexibility to the critical events. The weights are shown in red in Figure 4b. The time-distance diagrams of the resulting timetables  $\pi$  with  $\theta = 0.40$  and  $\theta = 0.95$  are shown in Figure 5a and 5b. In step 2 only one iteration was performed until the timetable was accepted. One can clearly see that here certain timetable events have much more flexibility than others. If we sum up the delay impacts of all events of the two scenarios  $\theta = 0.40$  and  $\theta = 0.95$ , respectively, we obtain an  $f_{sta}(\pi)$  -value reduced to 87.0% and 79.3%, respectively, compared to the one of step 1 (see Figure 6d).

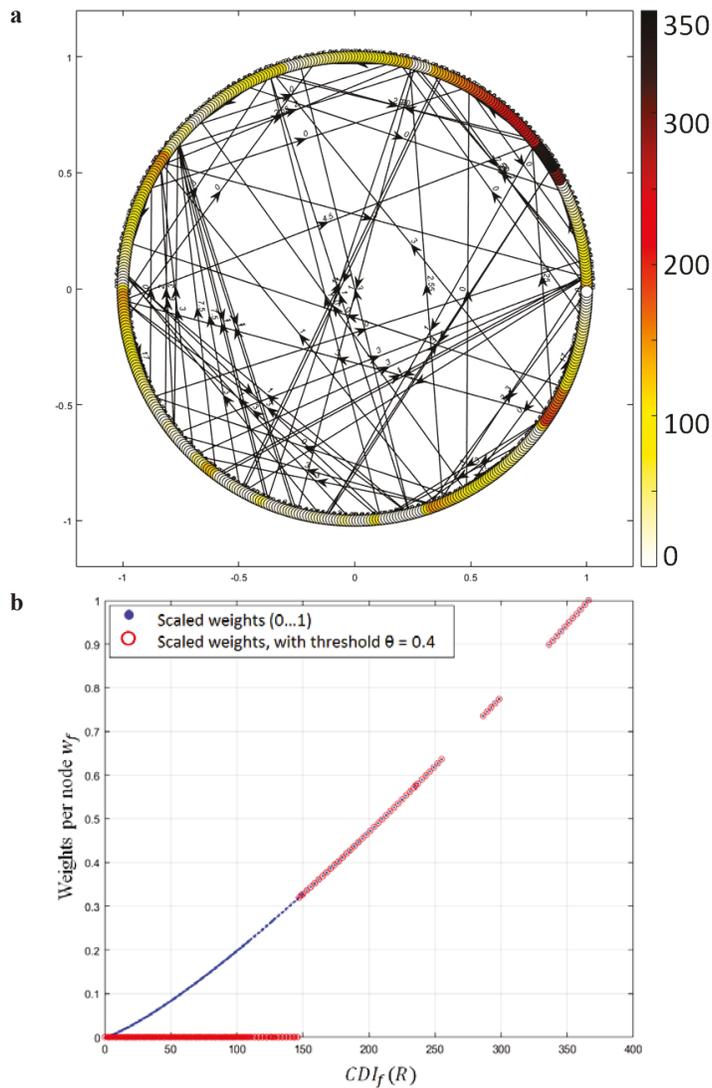


Figure 4: a) The values of the  $CDI_f(R)$  for all event nodes (1 to 500) of the timetable

after step 1 indicated by a colour code ranging from 0 (low impact) to 350 (high impact) and the interdependencies between the event nodes. b) shows the weights (calculated according to equation 10), normalized to values in the range of 0 to 1 for all event nodes.

**a:** Step 2: with threshold  $\theta = 0.95$

**b:** Step 2: with threshold  $\theta = 0.40$

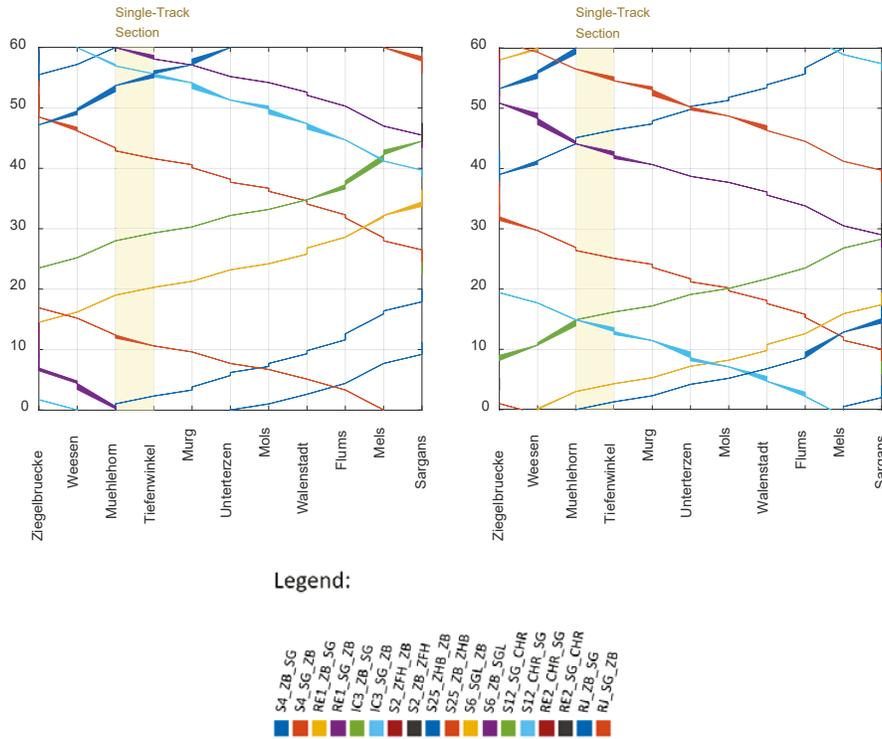


Figure 5: Time-distance diagram for the second iteration of the timetable calculation  $\pi$ . a) with a threshold  $\theta = 0.95$  and a resulting low number of weights  $w_f$  selected. b) with a threshold  $\theta = 0.40$  and a resulting rather high number of weights  $w_f$  selected. The line colours are the same as in Figure 3).

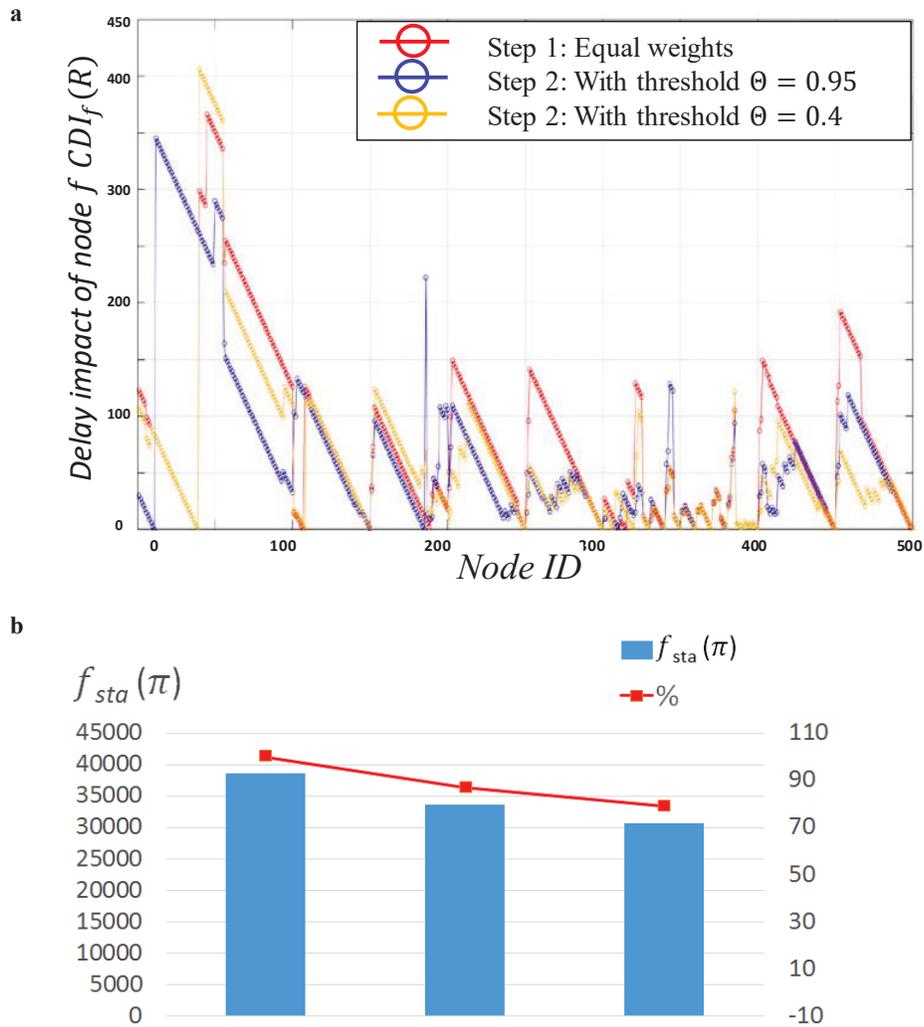


Figure 6: a) Distribution of the cumulative delay impact values  $CDI_f(R)$  across nodes  $f$  for timetable after step 1 with equal weights applied (red curve), and for timetable after step 2 with few selected weights (blue curve, see text for selection criteria) and after step 2 with all weights applied (orange curve). b) Improvement of step 2 for  $\theta = 0.95$  (middle bar) and for  $\theta = 0.4$  (right bar) relative to step 1 (left bar).

The  $f_{sta}(\pi)$ -value in Figure 6a and 6b indicated as ‘equal weights’ was calculated with weights equal to 1 for all events in step 1.

## 4 Conclusions

The aim of this research was to introduce an extension of FPESP with track selection and a heuristic improvement of solutions based on max-plus algebraic stability analysis. The track choice extension of the FPESP approach accounts for a mesoscopic infrastructure level of detail which is an additional requirement for generating operational timetable scenarios. Temporary changes of infrastructure properties like the number or the maximum allowed speed of tracks and switches reduce the available capacity for track assignments to train runs. For this reason, we introduce an extension of the FPESP model that we call ‘TCFPESP’ model. The TCFPESP model allows to make a target-oriented adjustment of event flexibility by applying weights to the TCFPESP objective function. We obtain those weights from the calculation of the cumulative delay impacts for all timetable events and use them in an iterative manner for improving timetable stability.

However the SI-based timetable calculation only results in feasible solutions of the TCFPESP model described in section 2.2 and a stability improvement by applying the iteration scheme presented in section 2.4 if the temporary restrictions of infrastructure properties are not too severe. If this is the case, the SI has to be relaxed (especially the functional requirements part of it). This requires eventually adaptations to the underlying line concept. This is a different use case than the one, that we described here. In Wüst et al. (2018a) we show, that using the SI for specifying the functional and non-functional input for maintenance timetabling, we can generate different timetables for different maintenance scenarios without having to change the functional part of the SI. This has the advantage, that communicating only earliest departure and latest arrival times, the commercial timetable can remain unchanged for the whole planning period. The complete application concept has been described in Wüst et al. (2018b).

With our results we demonstrate the operational benefit that can be obtained by utilizing the max-plus stability analysis for TCFPESP based re-planning. To make the effect of the iterative stability improvement more clear and to illustrate the interoperability between the TCFPESP and max-plus framework we do not apply event time constraints at the boundary nodes. This makes that the resulting timetables of each iteration differ significantly from each other as can be seen from Figure 5 a and b. It is however rather easy to add additional event time constraints in TCFPESP to force the solution of a successive stability improvement step be close to the previous one. As those additional constraints limit the range for the reduction of the  $CDI_f(R)$ , we did without them in the presented case study.

We show results for a few example scenarios which demonstrate that we can reduce the overall delay impact of timetable events by a significant amount (a reduction of more than 20% in the second iteration compared to the first iteration). We consider these preliminary results as promising for making target-oriented improvements of timetable stability, especially in cases where variability of process times is high and cannot be reduced by operational measures. Timetable events that have a strong influence on many other timetable events should be planned with more flexibility than those with low cumulated impact. On one side the use case that we selected is based on operational requirements and the mesoscopic data level for the scenario is characteristic for maintenance timetable planning. On the other side, we wanted to point out the strong impact of the stability analysis-based re-planning iterations. This is the reason, why we did not use time-dependencies to fix pass times at the boundary nodes of the corridor what would be more typical for the use case of maintenance timetable planning. In future

studies we would like to closer investigate the potential for stability improvement also under conditions when existing timetables must be altered. In this context, we also want to further investigate the presented observations with the help of simulations on microscopic level. Our aim is to develop more specific application rules for the presented framework.

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